PARAMETRIC AND NON-PARAMETRIC PRICING MODELS OF OPTIONS IN EUROPEAN MARKETS

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Annotation: This article explores European-style options, specifically the European call option, European put option, look-back call option, and look-back put option. The article also examines the payment models for these options. The main findings of the paper have theoretical significance and can be widely applied in financial mathematics. Furthermore, the constructed models can be used for option pricing calculations.

Keywords: option, parametric and non-parametric options, European-style options, alternative pricing, Black-Scholes model.

МОДЕЛИ ОЦЕНКИ ПАРАМЕТРИЧЕСКИХ И НЕПАРАМЕТРИЧЕСКИХ ОПЦИОНОВ ЕВРОПЕЙСКОГО ТИПА

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Аннотация: В данной статье исследуются опционы европейского типа. В частности, рассматриваются дискретные опционы на покупку и продажу европейского типа, а также опционы с оглядкой на стандарт на покупку и с оглядкой на стандарт на продажу, а также модели выплат для этих опционов.

Основные результаты статьи имеют теоретическое значение и могут быть широко применены в финансовой математике. Кроме того, построенные модели могут быть использованы для расчета цен опционов.

Ключевые слова: опцион, параметрические и непараметрические опционы, опционы европейского типа, альтернативное ценообразование, модель Блэка-Шоулза.

YEVROPA TURIDAGI PARAMETRIK VA NOPARAMETRIK OPSIONLARNI NARXLASH MODELLARI

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Annotatsiya: Mazkur maqolada Yevropa turidagi opsionlar o'rganiladi. Xususan, diskret holda Yevropa turidagi sotib olish (call option) opsioni, Yevropa turidagi sotish (put option) opsioni, oqibatli standart sotib olish (look back call option) opsioni, oqibatli standart sotish (look back put option) hamda bu opsionlar uchun to'lov modellari o'rganiladi.

Maqolada olingan asosiy natijalar nazariy ahamiyatga ega bo'lib, moliyaviy matematikada keng ko'lamli tatbiq qilish mumkin. Shuningdek ushbu qurilgan modellardan opsion narxlarini hisoblash uchun foydalansa bo'ladi.

Kalit so'zlar: opsion, parametrik va noparametrik opsionlar, Yevropa turidagi opsionlar , muqobil narx, Bleck-Sholes modeli.

INTRODUCTION

The rapid acceleration of economic growth in developed countries requires the modernization of their economies. This, in turn, necessitates the implementation of appropriate investment activity assessment models that ensure the realization of relevant innovative projects. This process is typically characterized by the availability of many alternatives for investing limited resources, which leads to the problem of determining the optimal direction for their use. As a financial instrument, selection today involves determining the alternative value of the future purchase or sale price of a product. The problem of determining alternative prices for options is of great importance today, as it helps prevent the emergence of arbitrage situations. Traditional methods applied to other financial instruments do not allow for the correct determination of option values, since the risk indicator for assets underlying options changes depending on the value and duration of the underlying assets. To solve this problem, several models have been developed. The Black-Scholes model is one of the first developed and remains a fundamental model.

LITERATURE REVIEW ON THE RESEARCH TOPIC

Information about European-style options can be found in the following article and books: Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy, 81*(3), 637-654. This paper introduces the Black-Scholes formula, which is widely used for option pricing and is one of the most important parametric models in financial economics. Hull, J. C. (2017). Options, Futures, and Other Derivatives (10th ed.). Pearson. Hull's book provides a comprehensive overview of financial derivatives, particularly options, futures, and other related financial instruments, including the Black-Scholes model and various other pricing methods. Jones, M. C., & McLachlan, G. J. (2007). Kernel Density Estimation. *Handbook of Mixture Analysis*, 389-400. This chapter discusses Kernel Density Estimation (KDE), a nonparametric method for estimating the probability density function, and its use in various statistical applications, including option pricing and others.

RESEARCH METHODOLOGY

This article addresses current issues in economics. Probability theory, mathematical statistics, and financial mathematics methods have been utilized in the research.

RESULTS AND DISCUSSION

The French mathematician L. Bachelier (1900) was the first to study option pricing as a dynamic, stochastic (random) process and to derive a formula for calculating the rational price in a complete market for a European option in continuous time. His model is considered the foundation in modern terms

$$S_t = S_0 + \mu * t + \sigma * W_t \ , \ 0 \le t \le T \qquad \textbf{(1)}$$

the equation is expressed in the following form, where: S_t - is the value of the option at time t, μ- is the drift coefficient, σ - is the volatility coefficient, W_t - is the standard Brownian motion, T- is the time remaining until maturity (in years). According to L. Bachelier's model, the rational price of a European call option is calculated using the following formula:

$$C_t = (S_0 - K)\Phi(\frac{S_0 - K}{\sigma * \sqrt{T}}) + \sigma * \sqrt{T}\phi(\frac{S_0 - K}{\sigma * \sqrt{T}}) \qquad \textbf{(2)}$$

Here, Φ and ϕ represent the standard normal distribution function and its probability density function, respectively. One of the drawbacks of Bachelier's model is that it allows for the possibility of negative option prices, which contradicts reality. Subsequent research showed that instead of considering the difference in option prices, one should consider the difference in their logarithms as independent.

In 1965, P. Samuelson studied the geometric (economic) Brownian motion to model stock price processes, which is expressed as follows:

$$S_t = S_0 * e^{\mu * t} * e^{\sigma * W_t - \sigma^2 * t/2} \quad \textbf{(3)}$$

Can you check if such a model is possible $dS_t = S_t * (\mu * dt + \sigma * dW_t)$ or $dS_t / S_t = (\mu * dt + \sigma * dW_t)$ it satisfies a differential equation of the given form. This model was thoroughly and comprehensively studied by R. Merton (1973), F. Black, and M. Scholes (1973), and is used to calculate the rational price of an option

$$C_t = e^{-rt/T} \cdot (S_0 \cdot e^{\mu t/T} \cdot \Phi(d_1) - K \cdot \Phi(d_2)) \quad \textbf{(4)}$$

$$d_1 = \frac{ln\frac{S_0}{K} + t/T * \left(\mu + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{t/T}},\ d_2 = \frac{ln\frac{S_0}{K} + t/T * \left(\mu - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{t/T}} \quad \textbf{(5)}$$

derived the formula. For this work, they were awarded the Nobel Prize in Economic Sciences in 1977

Note: In this formula, $e^{-rt/T}$ the coefficient, similar to the discrete case $C_N = (1+r)^{-N} Ef_N$, is akin to the continuous discounting coefficient. μ and σ the selection of parameters plays a key role in determining the future value of the underlying asset and in calculating the rational price of the option. The μ parameter is given differently depending on the type of the underlying asset:

- For non-dividend-paying stocks, μ = r (where r is the risk-free interest rate);
- For dividend-paying stocks with a dividend yield q, μ = r - q;
- For currency options, μ = r - q, where r is the risk-free interest rate and q is the base currency's interest rate.

- For an indexed stock option, μ = r - q, where q is the average dividend yield of the stock index over the life of the option contract.
- For a futures contract option $\mu = r$, S_0- is taken as the current futures price;
- For the option with a bond $\mu = r - q$, the coupon interest rate is given by q. If μ is not given in the problem statement, it is considered as $\mu = r$.

Example. In the Black-Scholes model K=100; S_0 =110; t=30 day; r=5%; σ= 0.27 (T(year)=365) find the alternative price for a European-style standard call option when: K- is the strike price, S_0- is the price of the option at the time of purchase, T- is the time remaining until the option expires, r -is the risk-free interest rate, and σ is the volatility coefficient

Solution: Calculate directly using the formula above:

$$C_t = e^{-0.5\times 30/365} \cdot (110 \cdot \Phi(1{,}3231) - 100 \cdot \Phi(1{,}2457)) = 10{,}79$$

$$d_1 = \frac{ln\frac{110}{100} + 30/365 * \left(0.05 + \frac{0.27^2}{2}\right)}{0.27\sqrt{30/365}} = 1.3231$$

$$d_2 = \frac{ln\frac{110}{100} + 30/365 * \left(0.05 - \frac{0.27^2}{2}\right)}{0.27\sqrt{30/365}} = 1.2457$$

Note: Calculation using Excel's financial function package There is no direct formula, but it can be calculated by providing the formula through Excel's function array.

Now let's focus on parametric and non-parametric option pricing models. The theoretical pricing of options is divided into two parts: parametric and non-parametric models. However, each of them has its advantages and limitations, and there is no clear evidence as to which one is preferred based on forecasting ability. Often, non-parametric methods are supported by the constantly changing volatility of the Black-Scholes model. We will examine the best-performing parametric models against non-parametric alternatives.

1. Parametric Option Pricing Models:

In a renowned paper, characteristic functions were introduced to obtain closed-form solutions for evaluating the Heston model. If the characteristic functions of the underlying probabilities are analytically known, these probabilities can be expressed through Fourier inversion, allowing closed-form solutions for option prices to be obtained.

In a risk-free economy, the price $S(t)$ of a non-dividend-paying stock and its components for any time t are given as follows:

$$\frac{dS(t)}{S(t)} = [\mathrm{R}(t) - \lambda\mu_J] + \sqrt{V(t)}d\omega_S(t) + \mathrm{J(t)dq(t)} \quad \textbf{(6)}$$

$$\mathrm{dV(t)} = [\theta_v - \kappa_v V(t)]dt + \sigma_v\sqrt{V(t)}d\omega_v(t) \quad \textbf{(7)}$$

$$\ln[1 + J(\mathrm{t})] \sim N\left(\ln[1+\mu_J] - \tfrac{1}{2}\sigma_J^2, \sigma_J^2\right) \quad \textbf{(8)}$$

In this context, $R(t)$ - is the instantaneous interest rate at time t, λ is the frequency of jumps per year, $V(t)$ - is the component of the variance of stock returns (given that no jump occurs), and $\omega_S(t)$ and $\omega_v(t)$ are standard Brownian motions, each equal to $\text{cov}\left[d\omega_s(t), \text{d}\omega_v(t)\right] = \rho dt$, The jump size J_s (given that a jump occurs) is always normally distributed, independent, and identically distributed with an unconditional mean μ_J and standard deviation (variance) $\ln\left[1 + J(\text{t})\right]$. The intensity of jumps, σ_J, follows a Poisson distribution with intensity q(t) and λ as the parameter. Here, $\Pr(\text{dq(t)} = 1) = \lambda dt$ and $\Pr(\text{dq(t)} = 0) = 1 - \lambda dt$, $\kappa_v, \theta_v / \kappa_v$, and σ_v represent, respectively, the rate of change, long-term mean, and diffusion variance of $V(t)$.The coefficient of variation of $V(t)$ is q(t), and $J(\text{t})$ is uncorrelated with each other, as well as uncorrelated with $\omega_S(t)$ and $\omega_v(t)$.

2. Nonparametric option pricing. The practical interest in the Black-Scholes pricing formula often arises from its analytical simplicity when determining the price of a European option on a non-dividend-paying asset.

This describes how the Black-Scholes model, which is widely used for pricing European options, is particularly attractive to practitioners due to its analytical simplicity, especially when applied to assets that do not pay dividends.

$$\text{C}_t = S_t N(d_1) - Ke^{-r\tau} N(d_2) \quad \textbf{(9)}$$

$$\text{d}_1 = \left[\ln(S_t / K) + (r + 0.5\sigma^2)\tau\right] / (\sigma\sqrt{\tau}) \text{ va } \text{d}_2 = d_1 - \sigma\sqrt{\tau}\,. \quad \textbf{(10)}$$

Here, N represents the joint normal distribution, S_t is the price of the underlying security, K is the strike price at the time the option contract is established, r-is the maturity time, and σ is the volatility of the underlying asset. Equation (9) does not take into account the preferences of individuals or the preferences of the overall market.

Nonparametric pricing models naturally extend, as it is easier to relax assumptions about the distribution. The natural nonparametric function for determining the price of a European call option on a non-dividend-paying asset links the option price to a set of variables that describe the option.

$$C_t = f\left(S_t, K, \sigma_t, \text{i}_t, \tau\right) \quad \textbf{(11)}$$

Here, S_t is the price of the underlying security, K is the price at the time the option is bought, *i*- is the interest rate, and τ is the maturity period. The unknown function f is then evaluated by a one-layer forward-propagating risk-neutral network, as described later.

CONCLUSION

Parametric models use specific parameters (such as volatility, risk-free rate) to describe the market when determining the option price. Black-Scholes and binomial models fall into this category.

Nonparametric models, on the other hand, do not rely on specific parameters to describe market conditions. Instead, they use historical data or stochastic processes for calculations. Monte Carlo simulation and Kernel Density Estimation (KDE) are examples of this approach.

The parameteric and nonparametric approaches to European option pricing have significantly evolved in recent years, leading to models that better capture the complexity of the market. The simplicity and efficiency of parametric models have made them widely used in practice, but they are only suitable for limited scenarios. On the other hand, nonparametric models allow for a broader and more realistic representation of the market, though they are more complex and computationally demanding. Therefore, combining both approaches and leveraging their strengths can provide the most effective method for pricing European options.

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