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BIR VA IKKI ARGUMENTLI FUNKSIYA UCHUN "AYNIGAN" YADROLI FREDGOLM TENGLAMALARI

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Annotasiya: Ikkala chegarasi o'zgaruvchi bo'lgan integral tenglamalar bo'lib ushbu mexanika, matematika fizika va texnikaning juda ko'plab masalalar ushbu ishda keltirilgan tenglamalar yordamida bajariladi.

Bunda Fredgolm tenglamasining xususiy bir holini ko`ramiz. Faraz qilaylik, Fredgolmning ikkinchi tur tenglamasi

$$u(x) = f(x) + \lambda \int_a^d K(x,t)u(t)dt \qquad (1)$$

berilgan.Agar bu tenglamada ishtirok etayotgan yadroni ushbu

$$K(x,t) = a_1(x)b_1(t) + a_2(x)b_2(t) + \ldots + a_n(x)b_n(t) \qquad (2)$$

ko`rinishda yozish mumkin bo`lsa, bunday yadro aynigan yadro deb yuritiladi[1]. bu holda (1) integral tenglamani chiziqli algebraic tenglamalar sistemasiga keltirib yechish mumkin.

Qisqaroq bayon qilish maqsadida n=3 deb olaylik. U holda (2) ifodani (1) tenglamaga qo`yib

$$u(x) = f(x) + \lambda \int_a^b [a_1(x)b_1(t) + a_2(x)b_2(t) + a_3(x)b_3(t)]u(t)dt$$

tenglamani hosil qilamiz; uni esa quyidagicha yozish mumkin:

$$u(x) = f(x) + \lambda a_1(x)\int_a^b b_1(t)u(t)dt + \lambda a_2(x)\int_a^b b_2(t)u(t)dt + \lambda a_3(x)\int_a^b b_3(t)u(t)dt\,. \qquad (3)$$

O`ng tomondagi aniq integrallar o`zgarmas sonlardan iborat bo`lib, ularni quyidagicha belgilab olamiz:

$$\int_a^b b_1(t)u(t)dt = Q_1\,, \quad \int_a^b b_2(t)u(t)dt = Q_2\,, \quad \int_a^b b_3(t)u(t)dt = Q_3\,. \qquad (4)$$

Bu integrallardagi u(t) funksiya noma'lum bo`lgani sababli , $Q_1$,$Q_2$ va $Q_3$ lar ham noma'lum sonlar bo`lib, ularni topish talab qilinadi. Shu maqsad bilan (4) ni (3) ga qo`yamiz:

$$u(x) = f(x) + \lambda Q_1 a_1(x) + \lambda Q_2 a_2(x) + \lambda Q_3 a_3(x)\,. \qquad (5)$$

Mana shu ifoda yordami bilan (4) tenglamalarning birinchisini o`zgartiramiz:

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$$Q_1 = \int_a^b b_1(t)u(t)dt = \int_a^b b_1(t)[f(t) + \lambda a_1(t)Q_1 + \lambda a_2(t)Q_2 + \lambda a_3(t)Q_3]dt =$$
$$= \int_a^b b_1(t)f(t)dt + \lambda Q_1 \int_a^b b_1(t)a_1(t)dt + \lambda Q_2 \int_a^b b_1(t)a_2(t)dt + \lambda Q_3 \int_a^b b_1(t)a_3(t)dt) \qquad (6)$$

O`ng tomondagi aniq integrallar o`zgarmas sonlar bo`ladi, ularni quyidagicha belgilab olamiz:

$$\int_a^b b_1(t)f(t)dt = A_1 \ , \qquad \int_a^b b_1(t)a_1(t)dt = \alpha_{11} \ ,$$

$$\int_a^b b_1(t)a_2(t)dt = \alpha_{12} \ , \qquad \int_a^b b_1(t)a_3(t)dt = \alpha_{13}$$

U holda (6) tenglik

$$Q_1 = A_1 + \lambda Q_1\alpha_{11} + \lambda Q_2\alpha_{12} + \lambda Q_3\alpha_{13}$$

Ko`rinishiga keladi. Bundagi $Q_1,Q_2,Q_3$ noma'lum sonlarni o`z ichiga oluvchi hadlarni tenglik ishorasining bir tomoniga o`tkazsak,

$$(1 - \lambda\alpha_{11})Q_1 - \lambda\alpha_{12}Q_2 - \lambda\alpha_{13}Q_3 = A_1$$

Uch noma'lumli chiziqli algebraic tenglama hosil bo`ladi.

Mana shunga o`xshash yana ikkita algebraic tenglamani keltirib chiqarish uchun (4) tenglamalarrning ikkinchi va uchinchisiga murojaat qilamiz:

$$Q_2 = \int_a^b b_2(t)u(t)dt = \int_a^b b_2(t)[f(t) + \lambda a_1(t)Q_1 + \lambda a_2(t)Q_2 + \lambda a_3(t)Q_3]dt =$$
$$= \int_a^b b_2(t)f(t)dt + \lambda Q_1 \int_a^b b_2(t)a_1(t)dt + \lambda Q_2 \int_a^b b_2(t)a_2(t)dt + \lambda Q_3 \int_a^b b_2(t)a_3(t)dt)$$

Bundagi integrallarni quyidagicha belgilaylik:

$$\int_a^b b_2(t)f(t)dt = A_2 \ , \qquad \int_a^b b_2(t)a_1(t)dt = \alpha_{21} \ ,$$

$$\int_a^b b_2(t)a_2(t)dt = \alpha_{22} \ , \qquad \int_a^b b_2(t)a_3(t)dt = \alpha_{23} \ .$$

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U holda

$$Q_2 = A_2 + \lambda Q_1\alpha_{21} + \lambda Q_2\alpha_{22} + \lambda Q_3\alpha_{23}$$

Yoki

$$(1-\lambda\alpha_{22})Q_2 - \lambda\alpha_{21}Q_1 - \lambda\alpha_{23}Q_3 = A_2$$

Hosil bo`ladi.

Xuddi shuningdek, (4) dan:

$$Q_3 = \int_a^b b_3(t)u(t)dt = \int_a^b b_3(t)[f(t) + \lambda a_1(t)Q_1 + \lambda a_2(t)Q_2 + \lambda a_3(t)Q_3]dt\ .$$

Buni ham yuqoridagilar kabi o`zgartirsak ushbu

$$(1-\lambda\alpha_{33})Q_3 - \lambda\alpha_{31}Q_1 - \lambda\alpha_{32}Q_2 = A_3$$

natija hosil bo`ladi; bunda

$$\int_a^b b_3(t)f(t)dt = A_3\ , \qquad \int_a^b b_3(t)a_1(t)dt = \alpha_{31}\ ,$$

$$\int_a^b b_3(t)a_2(t)dt = \alpha_{32}\ , \qquad \int_a^b b_3(t)a_3(t)dt = \alpha_{33}\ .$$

Shunday qilib, biz Q larga nisbatan quyidagi chiziqli algebraic tenglamalar sistemasini hosil qildik:

$$\begin{aligned}
&(1-\lambda\alpha_{11})Q_1 - \lambda\alpha_{12}Q_2 - \lambda\alpha_{13}Q_3 = A_1 \\
&-\lambda\alpha_{21}Q_1 + (1-\lambda\alpha_{22})Q_2 - \lambda\alpha_{23}Q_3 = A_2 \qquad (7) \\
&-\lambda\alpha_{31}Q_1 - \lambda\alpha_{32}Q_2 + (1-\lambda\alpha_{33})Q_3 = A_3
\end{aligned}$$

Bu sistemadagi A lar va α lar ma'lum sonlarrdir, chunki ulaga mos integrallar ishorasi orasidagi funksiyala masalada berilgan bo`ladi.

Endi (7) sistemani oily algebradagi Kramer fomulalari yodamida yechamiz:

$$Q_2 = \frac{D_1}{D}\ , \qquad Q_2 = \frac{D_2}{D}\ , \qquad Q_3 = \frac{D_3}{D}\ , \qquad (8)$$

Bu formulalarda

$$D=\begin{vmatrix}1-\lambda\alpha_{11} & -\lambda\alpha_{12} & -\lambda\alpha_{13}\\ -\lambda\alpha_{21} & 1-\lambda\alpha_{22} & -\lambda\alpha_{23}\\ -\lambda\alpha_{31} & -\lambda\alpha_{32} & 1-\lambda\alpha_{33}\end{vmatrix}. \qquad (9)$$

Ma'lumki, D_1 ni topish uchun (9) determinantda birinchi ustun elementlari o`rniga (7) dagi $A_1$,$A_2$,$A_3$ ozod hadlarni qo`yish kerak. D_2 va D_3 lar ham usulda topiladi. Shuni ham ta'kidlab o`tishimiz zarurki, (7) sistemadagi $A_1$,$A_2$ va $A_3$ larning kamida bittasi noldan farqli bo`lganda, (9) determinantning noldan farqli bo`lishi shart.

Demak, λ parametrning D determinantni nolga aylantirmaydigan hamma qiymalari uchun (2) ko`rinishdagi yadroli Fredgolm tenglamalarini shu usulda yechish mumkin ekan. Shubhasiz, bu masalada ishtiok etayogan barcha integrallar mavjud deb faraz qilinadi.

Ushbu tenglama yechilsin:

$$u(x)=x^2+\lambda\int_0^1(1+xt)u(t)dt\ .$$

Bu misoldagi λ parametr umumiy holda berilgan bo`lib, K(x,t)=1+xt yadro yuqoridagi (2) ko`rinishda ifodalangan. Tenglamaning o`ng tomonidagi integralni ikkiga ajratib,

$$\int_0^1(1+xt)u(t)dt=\int_0^1 u(t)dt+x\int_0^1 tu(t)dt\ ,$$

So`ngra quyidagicha

$$Q_1=\int_0^1 u(t)dt\ ,\qquad Q_2=\int_0^1 tu(t)dt$$

belgilaymiz. U holda berilgan integral tenglama

$$u(x)=x^2+\lambda Q_1+\lambda Q_2 x$$

Ko`rinishda yoziladi. Nama'lum funksiyaning mana shu ifodasidan foydalanib, Q_1 bilan Q_2 ni hisoblaymiz:

$$Q_1=\int_0^1 u(t)dt=\int_0^1(t^2+\lambda Q_1+\lambda Q_2 t)dt=\left.\frac{1}{3}t^3+\lambda Q_1 t+\frac{1}{2}\lambda Q_2 t^2\right|_0^1=\frac{1}{3}+\lambda Q_1+\frac{1}{2}\lambda Q_2\ ,$$

yoki

$$(1-\lambda)Q_1-\frac{1}{2}\lambda Q_2=\frac{1}{3}\ .$$

Xuddi shuningdek,

$$Q_2 = \int_0^1 tu(t)dt = \int_0^1 t(t^2 + \lambda Q_1 + \lambda Q_2 t)dt = \frac{1}{4} + \frac{1}{2}\lambda Q_1 + \frac{1}{3}\lambda Q_3$$

yoki

$$-\frac{1}{2}\lambda Q_1 + (1 - \frac{1}{3}\lambda)Q_2 = \frac{1}{4} \; .$$

Shunday qilib, quyidagi chiziqli algebraic tenglamalar sistemasi hosil bo`ldi:

$$(1-\lambda)Q_1 - \frac{1}{2}\lambda Q_2 = \frac{1}{3}$$

$$-\frac{1}{2}\lambda Q_1 + (1 - \frac{1}{3}\lambda)Q_2 = \frac{1}{4}$$

Bu sistemaning yechimi Kramer formulalariga asosan yozamiz:

$$Q_1 = \frac{D_1}{D} \; , \qquad Q_2 = \frac{D_2}{D}$$

bu yerda

$$D = \begin{vmatrix} 1-\lambda & -\frac{1}{2}\lambda \\ -\frac{1}{2}\lambda & 1-\frac{1}{3}\lambda \end{vmatrix} = \frac{1}{12}(\lambda^2 - 16\lambda + 12) \quad 0 \; ,$$

$$D_1 = \begin{vmatrix} \frac{1}{3} & -\frac{1}{2}\lambda \\ \frac{1}{4} & 1-\frac{1}{3}\lambda \end{vmatrix} = \frac{1}{72}(\lambda + 24) \; ,$$

$$D_2 = \begin{vmatrix} 1-\lambda & \frac{1}{3} \\ -\frac{1}{2}\lambda & \frac{1}{4} \end{vmatrix} = \frac{1}{12}(3-\lambda) \; .$$

Demak,

$$Q_1 = \frac{D_1}{D} = \frac{1}{6}\frac{\lambda + 24}{\lambda^2 - 16\lambda + 12} \; , \qquad Q_2 = \frac{D_2}{D} = \frac{3-\lambda}{\lambda^2 - 16\lambda + 12} \; .$$

Bularni izlanayogan noma'lum funksiyaning yuqoidagi ifodasiga qo`yib, uni quyidagi ko`rinishda yozamiz:

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$$u(x) = x^2 + \frac{\lambda(3-\lambda)}{\lambda^2 - 16\lambda + 12} x + \frac{\lambda(24+\lambda)}{6(\lambda^2 - 16\lambda + 12)} \, .$$

Bu esa berilgan masalaning yechimidi. Yechim ifodasidagi kasrlarning maxraji nolga teng bo`lmasligi uchun λ parametr

$$\lambda^2 + -16\lambda + 12 = 0$$

Kvadrat tenglamaning ildizi bo`lmasligi shart, ya'ni $\lambda \neq 8 \pm 2\sqrt{3}$. Xususiy holda λ =2 deb faraz qilsak, yechim quyidagicha yoziladi:

$$u(x) = x^2 - \frac{x}{8} - \frac{13}{24} \, .$$

Ushbu tenglama yechilsin:

$$u(x) = f(x) + \lambda \int_0^\pi \cos(x+t) u(t) dt \, .$$

Ma'lumki,

$$\cos(x+tt) = \cos x \cos t - \sin x \sin t \, ;$$

Demak, tenglamani

$$u(x) = f(x) + \lambda \cos x \int_0^\pi \cos t u(t) dt - \lambda \sin x \int_0^\pi \sin t u(t) dt = f(x) + \lambda \cos x Q_1 - \lambda \sin x Q_2$$

ko`rinishda yozish mumkin; bunda

$$Q_1 = \int_0^\pi \cos t u(t) dt \, , \qquad Q_2 = \int_0^\pi \sin t u(t) dt \, .$$

Bu integrallarda u(t) o`rniga uning yuqorida olingan ifodasini qo`yamiz:

$$Q_1 = \int_0^\pi \cos t [f(t) + \lambda \cos t Q_1 - \lambda \sin t Q_2] dt = \int_0^\pi \cos t f(t) dt + \lambda Q_1 \int_0^\pi \cos^2 t dt - \lambda Q_2 \int_0^\pi \cos t \sin t dt \, .$$

Integrallarning qiymatlari

$$\int_0^\pi \cos^2 t dt = \frac{\pi}{2} \qquad \int_0^\pi \cos t \sin t dt = 0$$

bo`lgani uchun birinchi tenglama

$$(1 - \frac{\lambda\pi}{2}) Q_1 = A \, ,$$

bo`ladi. Bu yerda

$$A = \int_0^{\pi} \cos t f(t)dt \,.$$

Xuddi shu usulda Q_2 ni izlaymiz:

$$Q_2 = \int_0^{\pi} \sin t[f(t) + \lambda \cos t Q_1 - \lambda \sin t Q_2]dt = \int_0^{\pi} \sin t f(t)dt + \lambda Q_1 \int_0^{\pi} \sin t \cos t dt - \lambda Q_2 \int_0^{\pi} \sin^2 t dt \,;$$

$$\int_0^{\pi} \sin^2 t dt = \frac{\pi}{2}$$

bo`lgani uchun

$$(1 + \frac{\lambda\pi}{2})Q_2 = B \,,$$

Bu yerda

$$B = \int_0^{\pi} \sin t f(t)dt \,.$$

Demak,

$$Q_1 = \frac{2}{2 - \lambda\pi} A \,, \qquad Q_2 = \frac{2}{2 + \lambda\pi} B \,.$$

Izlanayotgan yechim:

$$u(x) = f(x) + \frac{2\lambda \cos x}{2 - \lambda\pi} A - \frac{2\lambda \sin x}{2 + \lambda\pi} B \,.$$

Bu ifodadagi kasrlarning maxrajlari nolga aylanmasligi uchun $\lambda \neq \pm\frac{2}{\pi}$ bo`lishi kerak. Xususiy holda, agar $\lambda = 1, f(x) = x$ deb olsak,

$$A = \int_0^{\pi} t \cos t dt = -2 \,, \qquad B = \int_0^{\pi} t \sin t dt = \pi$$

bo`lib, yechim uchun quyidagi ifoda hosil bo`ladi:

$$u(x) = x - \frac{4}{2 - \pi} \cos x - \frac{2\pi}{2 + \pi} \sin x \,.$$

Ikki argumentli funksiya uchun

Agar ikki argumentli noma'lum funksiya uchun

$$u(x,y)=f(x,y)+\lambda\int_a^b\int_c^d K(x,y,t_1,t_2)u(t_1,t_2)dt_1dt_2 \quad (10)$$

Fredgolm tenglamasi berilgan bo`lib, uning yedrosi aynigan bo`lsa, ya'ni uni

$$K(x,y,t_1,t_2)=a_1(x,y)b_1(t_1,t_2)+a_2(x,y)b_2(t_1,t_2)+...+a_n(x,y)b_n(t_1,t_2) \quad (11)$$

Ko`rinishda yozish mumkin bo`lsa, bunday tenglamani yechish masalasini n ta chiziqli algebraic tenglamalar sistemasini yechishga keltirish mumkin. Buning qanday bajarilishini quyidagi misolda ko`rsatamiz.

Ushbu tenglama yechilsin:

$$u(x,y)=A(x+y)+\lambda\int_0^1\int_0^1 (y+t_2)u(t_1,t_2)dt_1dt_2\ ,$$

bu yerda A – o`zgarmasa son.

Berilgan tenglamani boshqacharoq yozib olaylik:

$$u(x,y)=A(x+y)+\lambda y\int_0^1\int_0^1 u(t_1,t_2)dt_1dt_2+\lambda\int_0^1\int_0^1 {}_2u(t_1,t_2)dt_1dt_2\ .$$

O`ng tomondagi aniq integrallar o`zgarmas sonlani beradi, shu sababli ularni

$$Q_1=\int_0^1\int_0^1 u(t_1,t_2)dt_1dt_2\ ,\qquad Q_2=\int_0^1\int_0^1 t_2u(t_1,t_2)dt_1dt_2$$

deb belgilab olamiz. U holda

$$u(x,y)=A(x+y)+\lambda Q_1y+\lambda Q_2\ .$$

Mana shu u ning ifodasini yuqorridagi integral ishoralari ostiga qo`yamiz, natijada

$$Q_1=\int_0^1\int_0^1[A(t_1+t_2)+\lambda Q_1t_2+\lambda Q_2]dt_1dt_2=A\int_0^1\int_0^1(t_1+t_2)dt_1\,dt_2+\lambda Q_1\int_0^1\int_0^1 t_2dt_1dt_2+\lambda Q_2\int_0^1\int_0^1 dt_1dt_2$$

bu integrallarni hisoblash natijasida

$$Q_1=A+\frac{1}{2}\lambda Q_1+\lambda Q_2\ ,\ \text{ya'ni}\ (1\text{-}\frac{\lambda}{2})Q_1-\lambda Q_2=A$$

kelib chiqadi. Xuddi shunga o`xshash,

$$Q_2=\int_0^1\int_0^1 t_2[A(t_1+t_2)+\lambda Qt_2+\lambda Q_2]dt_1dt_2=A\int_0^1\int_0^1 t_2(t_1+t_2)dt_1\,dt_2+\lambda Q_1\int_0^1\int_0^1 t_2^2dt_1dt_2+\lambda Q_2\int_0^1\int_0^1 dt_1dt_2$$

Bu integrallarni hisoblash naijasida

$$Q_2 = \frac{7}{12}A + \frac{1}{3}\lambda Q_1 + \frac{1}{2}\lambda Q_2,$$

ya'ni

$$-\frac{1}{3}\lambda Q_1 + (1-\frac{\lambda}{2})Q_2 = \frac{7}{12}A$$

kelib chiqadi. Shunday qilib, ushbu chiziqli tenglamalar sistemasi hosil bo`ldi:

$$(1-\frac{\lambda}{2})Q_1 - \lambda Q_2 = A,$$
$$-\frac{1}{3}\lambda Q_1 + (1-\frac{\lambda}{2})Q_2 = \frac{7}{12}A.$$

Kramer formulalariga muvofiq, bu sistemaning yechimi

$$Q_1 = \frac{D_{1`}}{D} = \frac{(12+\lambda)A}{12-12\lambda-\lambda^2}, \qquad Q_2 = \frac{D_{2`}}{D} = \frac{(14+\lambda)A}{2(12-12\lambda-\lambda^2)}.$$

U holda berilgan integral tenglamaning yechimi quyidagicha yoziladi:

$$u(x,y) = A(x+y) + \frac{\lambda(12+\lambda)A}{12-12\lambda-\lambda^2}y + \frac{\lambda(14+\lambda)A}{2(12-12\lambda-\lambda^2)},$$

Bunda $D \neq 0$, ya'ni $12-12\lambda-\lambda^2 \neq 0$.

Koeffitsientlarni tenglash usuli

Aynigan yadroli Fredgolm englamalarini boshqa bir usul bilan ham yechish mumkin. Bu usul esa mos koeffitsientlarni taqqoslashdan, ya'ni solishtiishdan iboratt. Biz bu usulni misollar yechish orqali ko`rsatish bilan chegaralanamiz.

Ushbu tenglamani yeching:

$$u(x) = \frac{5}{6}x - \frac{1}{9} + \frac{1}{3}\int_0^1 (x+t)u(t)dt.$$

O`ng tomondagi integralni quyidagicha belgilab olaylik:

$$\int_0^1 (x+t)u(t)d = x\int_0^1 u(t)dt + \int_0^1 tu(t)d = xQ_1 + Q_2.$$

U holda

$$u(x) = \frac{5}{6}x - \frac{1}{9} + \frac{1}{3}(Q_1x + Q_2) = (\frac{5}{6} + \frac{Q_1}{3})x + (\frac{Q_2}{3} - \frac{1}{9}) = \alpha x + \beta.$$

Bundagi α va β hozircha noma'lum sonlar. Endi u(x) ning so`nggi ifodasini berrilgan integral tenglamaga qo`yamiz:

$$\alpha x+\beta=\frac{5}{6}x-\frac{1}{9}+\frac{1}{3}\int_0^1(x+t)(\alpha t+\beta)dt.$$

O`ng tomondagi integralni hisoblab chiqilsa

$$\alpha x+\beta=(\frac{\alpha}{6}+\frac{\beta}{3}+\frac{5}{6})x+(\frac{\alpha}{9}+\frac{\beta}{9}-\frac{1}{9})$$

hosil bo`ladi. Bu tenglik ayniyat bo`lgani uchun, uning ikki tomonidagi x ning koeffitsientlari o`zaro va ozod hadlar ham o`zaro teng bo`lishi kerak. Ularni tenglash natijasida ushbu

$$\alpha=\frac{\alpha}{6}+\frac{\beta}{3}+\frac{5}{6}, \qquad \beta=\frac{\alpha}{9}+\frac{\beta}{6}-\frac{1}{9}$$

chiziqli algebraic tenglamalar sistemasi hosil bo`ladi. Ularni quyidagi ko`inishda yozish mumkin:

$$5\alpha-2\beta=5$$
$$2\alpha-15\beta=2$$

Bu sistemaning yechimi α =1, β =0 bo`ladi. Demak, berilgan integral tenglamaning yechimi

$$u(x)=\alpha x+\beta=x$$

bo`ladi.

Ushbu tenglamani yeching:

$$u(x,y)=\frac{xy}{2}-\frac{1}{3}+\int_0^1\int_0^1(xy+t_1t_2)u(t_1,t_2)dt_1dt_2.$$

O`ng tomondagi qavslarni ochib ikkala integralni ham qisqacha Q_1 va Q_2 orqali belgilaymiz:

$$u(x,y)=\frac{xy}{2}-\frac{1}{3}+xy\int_0^1\int_0^1 u(t_1,t_2)dt_1dt_2+\int_0^1\int_0^1 t_1t_2u(t_1,t_2)dt_1dt_2=\frac{xy}{2}-\frac{1}{3}+xyQ_1+Q_2=$$
$$=(Q_1+\frac{1}{2})xy+(Q_2-\frac{1}{3})=\alpha xy+\beta.$$

u ning mana shu ifodasini berilgan integral tenglamaga qo`yamiz:

$$\alpha xy+\beta=\frac{xy}{2}-\frac{1}{3}+\int_0^1\int_0^1(xy+t_1t_2)(\alpha t_1t_2+\beta)dt_1dt_2.$$

Bu yerdagi integrallar hisoblab chiqilsa, quyidagi ayniyat

$$\alpha xy + \beta = (\frac{1}{4}\alpha + \beta + \frac{1}{2})xy + (\frac{1}{9}\alpha + \frac{1}{4}\beta - \frac{1}{3}).$$

Hosil bo`ladi. Uning tomonidagi xy ning koeffitsientlarini o`zao hamda ozod hadlarni o`zaro tenglash natijasida quyidagi tenglamalar

$$\alpha = \frac{1}{4}\alpha + \beta + \frac{1}{2}, \qquad \beta = \frac{1}{9}\alpha + \frac{1}{4}\beta - \frac{1}{3},$$

ya’ni

$$\frac{3}{4}\alpha - \beta = \frac{1}{2},$$
$$\frac{1}{9}\alpha - \frac{3}{4}\beta = \frac{1}{3}$$

chiziqli algebraic tenglamalar sistemasi hosil bo`ladi. Bu sistemaning yechimi

$$\alpha = \frac{6}{65}, \qquad \beta = -\frac{28}{65}.$$

Demak, integral tenglamaning yechimi

$$u(x, y) = \alpha xy + \beta = \frac{6}{65}xy - \frac{28}{65}$$

bo`ladi.

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