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TEYLOR VA MAKLOREN QATORLARI. AYRIM FUNKSIYALARNING MAKLOREN QATORLARI.

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ANNOTATSIYA: Yig'indisi berilgan ixtiyoriy marta differensiallanuvchi funksiyaga teng bo'ladigan darajali qatorlarning mavjudligi va ularni topish masalasi Teylor va uning xususiy holi bo'lgan Makloren qatorlari yordamida o'rganiladi. Bunda berilgan funksiya bo'yicha tuzilgan darajali qatorning yaqinlashish oralig'ini topish va bu qator yig'indisini berilgan funksiyaga teng bo'lish shartlarini aniqlash masalalari qaraladi. Bunda Makloren qatorining Lagranj ko'rinishidagi qoldiq hadi muhim ahamiyatga ega bo'ladi. Asosiy elementar va ayrim elementar funksiyalarning Makloren qatorlari topilib, ularning yaqinlashish sohasi aniqlanadi.
Kalit so'zlar: Teylor qatori, Qoldiq had,Teylor qatorining yaqinlashishi,Qoldiq hadning Lagranj ko'rinishi ,Makloren qatori ,Binomial qator

1.Teylor va Makloren qatorlari. Ma'lumki berilgan ushbu

$$a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \cdots + a_n(x-x_0)^n + \cdots = \sum_{k=0} a_k(x-x_0)^k \quad (1)$$

darajali qatorning yig'indisi (x_0-R, x_0+R) yaqinlashish oralig'ida ixtiyoriy marta differensiallanuvchi biror $S(x)$ funksiyani aniqlaydi. Endi bu masalani teskarisini, ya'ni yig'indisi berilgan $f(x)$ funksiyaga teng bo'lgan (1) darajali qatorni topish masalasini qaraymiz. Albatta bunda $f(x)$ funksiya biror $x=x_0$ nuqta va uning qandaydir atrofida ixtiyoriy marta differensiallanuvchi deb hisoblanadi. Bu muammo juda ko'p nazariy va amaliy masalalarni yechishda paydo bo'ladi va ularning ayrimlarini keyinchalik ko'rib o'tamiz. Buning uchun $x=x_0$ nuqtaning biror atrofida

$$f(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \cdots + a_n(x-x_0)^n + \cdots \quad (2)$$

tenglik o'rinli deb faraz qilamiz. Bu tenglikdagi a_n (n=0,1,2,···) koeffitsiyentlarni topamiz. Dastlab (2) darajali qatorda $x=x_0$ deb $a_0=f(x_0)= f^{(0)}(x_0)$ ekanligini ko'ramiz. Endi (2) darajali qatorni hadlab differensiallab,

$$f'(x) = a_1 + 2a_2(x-x_0) + 3a_3(x-x_0)^2 + \cdots + na_n(x-x_0)^{n-1} + \cdots$$

tenglikka ega bo'lamiz va undan $a_1=f'(x_0)= f^{(1)}(x_0)$ natijani olamiz. Oxirgi darajali qatorni yana bir marta differensiallab,

$$f''(x) = 2\cdot 1a_2 + 3\cdot 2a_3(x-x_0) + \cdots + n(n-1)a_n(x-x_0)^{n-2} + \cdots$$

darajali qatorni hosil etamiz va unda $x=x_0$ deb $a_2=f''(x_0)/(2\cdot1)= f^{(2)}(x_0)/2!$ ekanligini ko'ramiz. Bu jarayonni davom ettirib, (2) darajali qator koeffitsiyentlari uchun

$$a_n = \frac{f^{(n)}(x_0)}{n!}, \quad n = 0,1,2,\cdots \quad (3)$$

formulani hosil qilamiz.

(3) formula orqali topiladigan a_n koeffitsiyentlardan foydalanib, ushbu darajali qatorni hosil etamiz:

$$f(x_0)+\frac{f\ (x_0)}{1!}(x-x_0)+\frac{f\ \ (x_0)}{2!}(x-x_0)^2+\cdots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+\cdots . \quad (3)$$

1-TA'RIF: (3) darajali qator $f(x)$ funksiya uchun ***Teylor qatori*** deb ataladi.

Shuni ta'kidlab o'tish kerakki, (3) qatorga o'xshash qatorlar dastlab 1694 yilda shveytsariyalik buyuk matematik I. Bernulli tomonidan qaralgan, ammo (3) ko'rinishda ingliz matematigi B.Teylor (1685–1731 y.) tomonidan 1812 yilda chop etilgan.

Berilgan $f(x)$ bo'yicha hosil qilingan (3) Teylor qatorini qarayotganimizda quyidagi uch hol bo'lishi mumkin:

- (3) darajali qator $x=x_0$ nuqtadan boshqa barcha nuqtalarda uzoqlashuvchi ;
- (3) qator yaqinlashuvchi, ammo uning yig'indisi berilgan $f(x)$ funksiyadan farqli boshqa bir funksiyadan iborat. Bunga misol sifatida

$$f(x)=\begin{matrix} e^{-1/x^2}, & x\ \ 0, \\ 0, & x=0 \end{matrix} \quad (4)$$

funksiyani qaraymiz. Bu funksiya ixtiyoriy marta differensiallanuvchi va uning barcha hosilalari $x_0=0$ nuqtada $f^{(n)}(0)=0$ ($n=0,1,2,\cdots$) shartni qanoatlantirishini ko'rsatish mumkin. Shu sababli (4) funksiyaning Teylor qatori $\underset{k=0}{0\ x^k}$ ko'rinishda bo'lib, uning yig'indisi $S(x)=0\neq f(x)$ funksiyadan iboratdir;

- (3) qator yaqinlashuvchi va uning yig'indisi berilgan $f(x)$ funksiyaga teng .

Biz uchun oxirgi hol bo'lishi maqsadga muvofiq va buning uchun $f(x)$ funksiya qanday shartni qanoatlantirishi kerakligini aniqlaymiz. Bu maqsadda $f(x)$ funksiya va uning (3) Teylor qatori bo'yicha hosil qilingan ushbu funksiyani qaraymiz:

$$R_n(x)=f(x)-\sum_{k=0}^{n}\frac{f^{(k)}(x_0)}{k!}(x-x_0)^k . \quad (5)$$

2-TA'RIF: (5) funksiya $f(x)$ funksiya Teylor qatorining ***n-qoldiq hadi*** deyiladi.

(3) va (5) tengliklardan bevosita quyidagi teorema kelib chiqadi:

1-TEOREMA: Berilgan $f(x)$ funksiyaning (3) Teylor qatori $x=x_0$ nuqtaning biror atrofida yaqinlashuvchi va uning yig'indisi $f(x)$ funksiyaga teng bo'lishi uchun uning (5) qoldiq hadi shu atrofdagi barcha x nuqtalarda

$$\lim_{n\to} R_n(x)=0 \quad (6)$$

shartni qanoatlantirishi zarur va yetarlidir.

Shunday qilib, (6) shart bajarilganda

$$f(x)=f(x_0)+\frac{f\ (x_0)}{1!}(x-x_0)+\frac{f\ \ (x_0)}{2!}(x-x_0)^2+\cdots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+\cdots$$

yoki , qisqacha qilib yozganda,

$$f(x)=\sum_{k=0}\frac{f^{(k)}(x_0)}{k!}(x-x_0)^k \quad (7)$$

tenglik o'rinli bo'ladi.

Agar $f(x)$ funksiya (1) ko'rinishdagi biror darajali qatorga yoyilsa, bu qator albatta (7) Teylor qatoridan iborat bo'lishi tushunarlidir. Bundan $f(x)$ funksiya darajali qatorga yoyilsa, bu qator yagona ravishda aniqlanishi kelib chiqadi.

(6) shartni bevosita tekshirish qiyin va shu sababli Teylor qatorining (5) qoldiq hadini

$$R_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-x_0)^{n+1}, \quad c\in(x_0,x) \qquad (8)$$

ko‘rinishda yozish mumkinligidan foydalanamiz (bu tasdiqni isbotsiz qabul etamiz).

3-TA’RIF: (8) tenglik $f(x)$ funksiya uchun Teylor qatorining ***Lagranj ko‘rinishidagi n-qoldiq hadi*** deyiladi.

Teylor qatorining Lagrang ko‘rinishidagi (8) qoldiq hadidan foydalanib, (6) shart bajarilishi uchun yetarli shartni topamiz.

2-TEOREMA: Agar $f(x)$ funksiya va uning hosilalari biror $[x_0-\alpha,\ x_0+\alpha]$ kesmada yuqoridan bir xil son bilan chegaralangan, ya’ni biror musbat M soni uchun

$$\left|f^{(n)}(x)\right|\le M\ (n=0,1,2,\cdots), \quad x\in[x_0-\alpha, x_0+\alpha], \qquad (9)$$

tengsizliklar o‘rinli bo‘lsa, unda (6) shart bajariladi .

Isbot: (9) shart bajarilganda, (8) formulaga asosan, (5) qoldiq hadni quyidagicha baholash mumkin:

$$\left|R_n(x)\right|=\frac{\left|f^{(n+1)}(c)\right|}{(n+1)!}\left|x-x_0\right|^{n+1}\le\frac{M}{(n+1)!}\left|x-x_0\right|^{n+1}\le\frac{M}{(n+1)!}\alpha^{n+1}.$$

Bu yerdan (6) shart bajarilishi uchun

$$\lim_{n\to\infty}\frac{\alpha^n}{n!}=0 \qquad (10)$$

ekanligini ko‘rsatish kifoya. Agar $0\le\alpha\le1$ bo‘lsa, (10) tenglik bajarilishi ravshan va shu sababli $\alpha>1$ holni qarash yetarli. Bu holda $u_n=\alpha^n/n!$ deb belgilasak, unda

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\lim_{n\to\infty}\frac{\alpha}{n+1}=0$$

tenglik o‘rinli bo‘ladi. Bu yerdan biror N soni uchun $n>N$ bo‘lganda $u_{n+1}< u_n$, ya’ni u_n monoton kamayuvchi ketma-ketlik ekanligini ko‘ramiz. Bundan tashqari $u_n>0$, ya’ni bu ketma-ketlik quyidan chegaralangan. Shu sababli monoton ketma-ketlik limiti haqidagi teoremaga asosan $\lim_{n\to\infty}u_n=A\ge0$ limit mavjud . Bu holda

$$A=\lim_{n\to\infty}u_{n+1}=\lim_{n\to\infty}(u_n\cdot\frac{\alpha}{n+1})=\lim_{n\to\infty}u_n\cdot\lim_{n\to\infty}\frac{\alpha}{n+1}=A\cdot0=0.$$

Demak, haqiqatan ham (10) tenglik o‘rinli va shu sababli (6) shart bajariladi.

Odatda Teylor qatorida $x_0=0$ bo‘lgan hol, ya’ni

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\cdots+\frac{f^{(n)}(0)}{n!}x^n+\cdots=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k \qquad (11)$$

darajali qator keng qo‘llaniladi.

4-TA’RIF: (11) darajali qator $f(x)$ funksiyaning ***Makloren qatori*** deb ataladi.

Makloren qatori uchun qoldir hadning Lagranj ko‘rinishi quyidagicha bo‘ladi:

$$R_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}, \quad c\in(0,x) .$$

2. **Ayrim funksiyalarning Makloren qatorlari .** Dastlab bir nechta $f(x)$ elementar funksiyalar uchun Makloren qatorlarini yozib, ularning yaqinlashish sohasini va berilgan $f(x)$ funksiyaga yaqinlashuvini tekshiramiz.

- $f(x)$=sin$x$. Bu funksiya uchun ixtiyoriy tartibli hosila mavjud va ularni birin-ketin topamiz:

$$f\ (x)=\cos x\,,\ f\ \ (x)=-\sin x\,,\quad f\ \ \ (x)=-\cos x\,,\quad f^{(4)}(x)=\sin x\,,\cdots,$$

$$f^{(n+4)}(x)=f^{(n)}(x)\,,\quad n=0,1,2,\cdots\ .$$

Bu yerdan quyidagi tengliklarga ega bo‘lamiz:

$$f(0)=,\ f\ (0)=1\,,\quad f\ \ (0)=0\,,\quad f\ \ \ (0)=-1\,,\quad f^{(4)}(0)=0\,,\cdots,$$

$$f^{(2n)}(0)=0\,,\quad f^{(2n+1)}(0)=(-1)^{n}\ .$$

$f(x)$=sin$x$ funksiya Makloren qatorining qoldiq hadini baholash uchun uning hosilalarini, keltirish formulalariga asosan,

$$f^{(n)}(x)=\sin(x+\frac{\pi}{2}n)$$

ko‘rinishda yozish mumkinligidan foydalanamiz. Bu yerdan ixtiyoriy x uchun $|f^{(n)}(x)|\leq 1$ ekanligi kelib chiqadi. Demak, 2-teoremaga asosan, $f(x)$=sin$x$ funksiyaning Makloren qatori $(-\infty, \infty)$ oraliqda yaqinlashuvchi va uning yig‘indisi shu funksiyaning o‘ziga teng, ya’ni

$$\sin x=x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}-\frac{x^{7}}{7!}+\cdots+(-1)^{n+1}\frac{x^{2n-1}}{(2n-1)!}+\cdots=\underset{k=0}{}(-1)^{k+1}\frac{x^{2k+1}}{(2k+1)!}\ . \quad (12)$$

- $f(x)$=cos$x$. Bu funksiya uchun ham ixtiyoriy tartibli hosila mavjud va ular

$$f\ (x)=-\sin x\,,\ f\ \ (x)=-\cos x\,,\quad f\ \ \ (x)=\sin x\,,\quad f^{(4)}(x)=\cos x\,,\cdots,$$

$$f^{(n+4)}(x)=f^{(n)}(x)\,,\quad n=0,1,2,\cdots$$

tengliklar bilan aniqlanadi. Bu yerdan quyidagi tengliklarga ega bo‘lamiz:

$$f(0)=,\ f\ (0)=1\,,\quad f\ \ (0)=0\,,\quad f\ \ \ (0)=-1\,,\quad f^{(4)}(0)=0\,,\cdots,$$

$$f^{(2n)}(0)=0\,,\quad f^{(2n+1)}(0)=(-1)^{n}\ .$$

$f(x)$=cos$x$ funksiya uchun ham uning hosilalarini

$$f^{(n)}(x)=\cos(x+\frac{\pi}{2}n)$$

ko‘rinishda yozish mumkinligidan foydalanib, ixtiyoriy x uchun $|f^{(n)}(x)|\leq 1$ ekanligini ko‘ramiz. Demak, 2-teoremaga asosan, $f(x)$=cos$x$ funksiyaning Makloren qatori $(-\infty, \infty)$ oraliqda yaqinlashuvchi va

$$\cos x=1-\frac{x^{2}}{2!}+\frac{x^{4}}{4!}-\frac{x^{6}}{6!}+\cdots+(-1)^{n}\frac{x^{2n}}{(2n)!}+\cdots=\underset{k=0}{}(-1)^{k}\frac{x^{2k}}{(2k)!} \quad (13)$$

tenglik o‘rinlidir.

- $f(x)=e^{x}$. Bu funksiyaning ixtiyoriy tartibli hosilasi mavjud va $f^{(n)}(x)=e^{x}$ va $f^{(n)}(0)=1$ (n=0,1,2,⋯) bo‘ladi. Bundan tashqari, ixtiyoriy A musbat soni uchun $[-A, A]$ kesmada $f^{(n)}(x)<e^{A}$ (n=0,1,2,⋯), ya’ni (9) shart bajariladi. Bulardan, $f(x)=e^{x}$ funksiyaning Makloren qatori $(-\infty, \infty)$ oraliqda yaqinlashuvchi va

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^{2}}{2!} + \cdots + \frac{x^{n}}{n!} + \cdots = \sum_{k=0} \frac{x^{k}}{k!} \qquad (14)$$

ko‘rinishda bo‘lishi kelib chiqadi.

- $f(x)$=sh$x$. ***Giperbolik sinus*** deb ataladigan bu funksiyaning Makloren qatorini topish uchun dastlab (14) qatorda x o‘zgaruvchini $-x$ bilan almashtiramiz:

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^{2}}{2!} - \frac{x^{3}}{3!} + \cdots + (-1)^{n}\frac{x^{n}}{n!} + \cdots = \sum_{k=0} (-1)^{k}\frac{x^{k}}{k!} \, . \qquad (15)$$

(14) va (15) Makloren qatorlarni hadma-had ayirish orqali $(-\infty, \infty)$ oraliqda o‘rinli bo‘lgan

$$\text{sh}x = \frac{e^{x} - e^{-x}}{2} = x + \frac{x^{3}}{3!} + \frac{x^{5}}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{k=0} \frac{x^{2k+1}}{(2k+1)!} \qquad (16)$$

natijani olamiz.

- $f(x)$=ch$x$. ***Giperbolik kosinus*** deb ataladigan bu funksiyaning Makloren qatorini topish uchun (14) va (15) qatorlarni hadlab qo‘shamiz:

$$\text{ch}x = \frac{e^{x} + e^{-x}}{2} = 1 + \frac{x^{2}}{2!} + \frac{x^{4}}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots = \sum_{k=0} \frac{x^{2k}}{(2k)!} \, . \qquad (17)$$

Bu qatorning ham yaqinlashish oralig‘i $(-\infty, \infty)$ bo‘ladi.

- $f(x)=(1+x)^{\alpha}$. Bunda α- ixtiyoriy o‘zgarmas sonni ifodalaydi. Bu funksiyaning hosilalarini topamiz:

$$f\ (x) = \alpha(1+x)^{\alpha-1}, \quad f\ \ (x) = \alpha(\alpha-1)(1+x)^{\alpha-2}, \ \cdots \ ,$$

$$f^{(m)}(x) = \alpha(\alpha-1)(\alpha-2)\cdots(\alpha-m+1)(1+x)^{\alpha-m}, \quad m = 0,1,2,\cdots .$$

Berilgan $f(x)=(1+x)^{\alpha}$ funksiyaning Makloren qatori $(-1,1)$ oraliqda yaqinlashuvchi va uning yig‘indisi funksiyani o‘ziga teng bo‘lishini ko‘rsatish mumkin, ya’ni

$$(1+x)^{\alpha} = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^{2} + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^{n} + \cdots =$$

$$= \sum_{k=0} \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!}x^{k} \qquad (18)$$

munosabat o‘rinli bo‘ladi. Bu darajali qator ***binomial qator*** deb ataladi.

Izoh: Agar (15) qatorda $\alpha=n=1,2,3, \cdots$, ya’ni natural songa teng bo‘lsa, unda $m>n$ holda $f^{(m)}(x)=0$ bo‘ladi. Natijada (15) qator chekli yig‘indiga aylanib, undan

$$(1+x)^{n} = \sum_{k=0}^{n} C_{n}^{k}x^{k} \, , \quad C_{n}^{k} = \frac{n!}{k!(n-k)!} = \frac{n\ \ (n-1)\cdots(n-k+1)}{k!},$$

ya’ni Nyuton binomi (I bob, §3, (5) ga qarang) kelib chiqadi.

Binomial qatorning kelgusida qo‘llaniladigan ikkita xususiy holini qaraymiz:

$$(1+x)^{-1} = \frac{1}{1+x} = 1 - x + x^{2} - x^{3} + \cdots + (-1)^{n+1}x^{n} + \cdots = \sum_{k=0} (-1)^{n}x^{k} \, ; \quad (19)$$

$$(1+x)^{-1/2} = \frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\ \ 3}{2\ \ 4}x^{2} - \frac{1\ \ 3\ \ 5}{2\ \ 4\ \ 6}x^{3} + \cdots +$$

$$+(-1)^n \frac{1\ 3\ \ldots\ (2n-1)}{2\ 4\ \ldots\ 2n} x^n + \cdots = \sum_{k=0} (-1)^k \frac{(2k-1)!!}{(2k)!!} x^k \ . \qquad (20)$$

(20) Makloren qatorida (2*k*–1)!! belgi 2*k*–1 va ungacha bo‘lgan toq sonlar, (2*k*)!! esa 2*k* va ungacha bo‘lgan juft sonlar ko‘paytmasini ifodalaydi .

- *f*(*x*)=ln(1+*x*). (19) qatorda *x* o‘zgaruvchini *t* bilan almashtirib va bu

qatorni (0, *x*) oraliqda (|*x*|<1) hadlab integrallab, ushbu Makloren qatorini hosil etamiz:

$$\ln(1+x) = \int_0^x \frac{dt}{1+t} = \int_0^x \sum_{k=0} (-1)^k t^k dt = \sum_{k=0} (-1)^k \int_0^x t^k dt = \sum_{k=0} (-1)^k \frac{x^{k+1}}{k+1} \ . \quad (21)$$

(21) darajali qator sifatida *x*=1 nuqtada ham yaqinlashuvchi bo‘lishi oldin (§4, (16) ga qarang) ko‘rsatilgan edi. Endi *x*=1 nuqtada bu qatorning yig‘indisi *S*=ln2 bo‘lishini, ya’ni

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + (-1)^{n+1}\frac{1}{n+1} + \cdots = \sum_{k=0} (-1)^{k+1}\frac{1}{k+1} \qquad (22)$$

tenglik o‘rinli ekanligini ko‘rsatamiz. Buning uchun

$$\frac{1}{1+x} \quad 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + (-1)^{n+1}\frac{x^{n+1}}{1+x}$$

ayniyatni [0, 1] kesma bo‘yicha hadlab integrallaymiz:

$$\int_0^1 \frac{dx}{1+x} = \int_0^1 dx - \int_0^1 xdx + \int_0^1 x^2 dx - \int_0^1 x^3 dx + \cdots + (-1)^n \int_0^1 x^n dx + (-1)^{n+1}\int_0^1 \frac{x^{n+1}}{1+x} dx$$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + (-1)^n \frac{1}{n+1} + R_n = S_n + R_n \ .$$

Bu yerdan (22) tenglik quyidagicha keltirib chiqariladi:

$$|R_n| = \left|\int_0^1 \frac{x^{n+1}}{1+x} dx\right| \quad \int_0^1 x^{n+1} dx = \frac{1}{n+2} \qquad \lim_{n\to} R_n = 0 \qquad \lim_{n\to} S_n = \ln 2 .$$

Demak, (21) Makloren qatorining yaqinlashish sohasi (–1,1] yarim oraliqdan iboratdir.

- *f*(*x*)=arctg*x* . (19) darajali qatorda *x* o‘zgaruvchini t^2 bilan almashtirib va bu qatorni (0, *x*) oraliqda (|*x*|<1) hadlab integrallab,

$$\text{arctg}x = \int_0^x \frac{dt}{1+t^2} = \int_0^x \sum_{k=0} (-1)^k t^{2k} dt = \sum_{k=0} (-1)^k \int_0^x t^{2k} dt = \sum_{k=0} (-1)^k \frac{x^{2k+1}}{2k+1} \qquad (23)$$

Makloren qatoriga ega bo‘lamiz. Bu darajali qator *x*=±1 chegaraviy nuqtalarda Leybnits shartlarini qanoatlantiruvchi va shu sababli yaqinlashuvchi bo‘lgan ishorasi navbatlanuvchi sonli qatorga aylanadi. Yuqoridagiga o‘xshab, *x*=±1 bo‘lganda uning yig‘indisi *S*=arctg(±1)= ±π/4 bo‘lishini ko‘rsatish mumkin. Shu sababli (23) Makloren qatorining yaqinlashish sohasi [–1, 1] kesmadan iboratdir.

- *f*(*x*)=arcsin*x* . (20) darajali qatorda *x* o‘zgaruvchini $-t^2$ bilan almashtirib va hosil bo‘lgan qatorni (0, *x*) oraliqda (|*x*|<1) hadlab integrallab, berilgan funksiyaning ushbu

$$\arcsin x = \int_0^x \frac{dt}{\sqrt{1-t^2}} = \int_0^x \sum_{k=0} \frac{(2k-1)!!}{(2k)!!} t^{2k} dt = \sum_{k=0} \frac{(2k-1)!!}{(2k)!!} \int_0^x t^{2k} dt =$$

$$=\sum_{k=0}\frac{(2k-1)!!}{(2k)!!}\frac{x^{2k+1}}{2k+1} \qquad (24)$$

Makloren qatorini hosil qilamiz. Bu qator $x=\pm 1$ nuqtalarda ham yaqinlashuvchi va yig‘indisi arcsin(±1)= ±π/2 bo‘lishini ko‘rsatish mumkin. Demak, (24) Makloren qatorining yaqinlashish sohasi [–1, 1] kesmadan iboratdir.

Bu qatorlardan foydalanib boshqa funksiyalarning Makloren qatorlarini topish mumkin. Misol sifatida $f(x)=\cos^2 x$ funksiyaning Makloren qatorini aniqlaymiz. Buning uchun uni

$$\cos^2 x=\frac{1+\cos 2x}{2}=\frac{1}{2}(1+\cos 2x) \qquad (25)$$

ko‘rinishda yozamiz. Endi $y=\cos x$ funksiyaning (13) Makloren qatorida x o‘zgaruvchini $2x$ bilan almashtirib, $y=\cos 2x$ funksiya Makloren qatorini hosil etamiz:

$$\cos 2x=1-\frac{2^2x^2}{2!}+\frac{2^4x^4}{4!}-\frac{2^6x^6}{6!}+\cdots+(-1)^n\frac{2^{2n}x^{2n}}{(2n)!}+\cdots=\sum_{k=0}(-1)^k\frac{2^{2k}x^{2k}}{(2k)!}.$$

Bu natijani (25) tenglikka qo‘yib, berilgan funksiyaning Makloren qatoriga ega bo‘lamiz:

$$\cos^2 x=\frac{1}{2}(1+\cos 2x)=1-\frac{2x^2}{2!}+\frac{2^3x^4}{4!}-\frac{2^5x^6}{6!}+\cdots+(-1)^n\frac{2^{2n-1}x^{2n}}{(2n)!}+\cdots$$

Foydalanilgan adabiyotlar ro’yhati.

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