

## RATSIONAL KASRLAR VA ULARNI INTEGRALLASHGA DOIR BA'ZI MASALALAR.

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**ANNOTATSIYA:** Har qanday aniqmas integral elementar funksiyalar orqali ifodalanishi shart emas. Shu sababli elementar funksiyalarda ifodalanadigan integrallar sinfini topish masalasi paydo bo'ladi. Bu masalaning xususiy bir javobi sifatida ratsional funksiyalardan olingan integrallarni ko'rsatish mumkin. Bunda dastlab I-IV turdagi eng sodda ratsional funksiyalarni integrallash usuli ko'rsatiladi. So'ngra, ixtiyoriy ratsional funksiyani ularning algebraik yig'indisi kabi yozish mumkinligidan foydalanib, umumiy holda ratsional funksiyadan olingan integrallarni hisoblash amalga oshiriladi. Bu integrallar logarifmik,  $\arctg(ax+b)$  ko'rinishdagi teskari trigonometrik funksiyalar hamda ratsional kasrlar, ya'ni elementar funksiyalar orqali ifodalanadi.

**Kalit so'zlar:** Ko'phad \* Ratsional kasr (funksiya) \* Noto'g'ri ratsional kasr \* To'g'ri ratsional kasr \* I tur eng sodda ratsional kasr \* II tur eng sodda ratsional kasr

**1.Ratsional funksiyalar.** Ma'lumki ,

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0 \quad (a_n \neq 0) \quad (1)$$

ko'rinishdagi funksiya **ko'phad** deyiladi. Bunda  $a_n, a_{n-1}, a_{n-2}, \dots, a_1, a_0$  o'zgarimas sonlar bo'lib, ular ko'phadning **ko'effitsiyentlari** ,  $n$  esa ko'phadning **darajasi** deb ataladi.

Masalan,  $P_3(x) = 5x^3 - x^2 + 2x + 4$  – III darajali,  $P_2(x) = 3x^2 - 5x + 2$  – II darajali,  $P_1(x) = 8x + 3$  – I darajali ko'phadlardir.

**Izoh:** Har qanday o'zgarimas funksiyani  $P_0(x) = a_0$  – 0-darajali ko'phad deb qarash mumkin.

**1-TA'RIF:** Ikkita ko'phad nisbatidan iborat funksiya **ratsional kasr yoki ratsional funksiya** deyiladi.

Odatda ratsional kasr  $R(x)$  kabi belgilanadi va, ta'rifga asosan,

$$R(x) = \frac{Q_m(x)}{P_n(x)} = \frac{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0}{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0} \quad (2)$$

ko'rinishda bo'ladi.

Masalan,

$$\frac{3x - 5}{x^2 - 2x + 7}, \quad \frac{4x^2 + x - 3}{5x^2 - 3x + 1}, \quad \frac{6x^3 + 5x^2 + 9x - 3}{5x^2 - 3x + 1}$$

ratsional kasrlardir.

**Izoh:** Har qanday  $Q_m(x)$  ko'phadni maxraji  $P_0(x) = 1$  bo'lgan ratsional kasr kabi qarash mumkin va shu nuqtai nazardan ko'phadlar ba'zan butun funksiyalar deb ataladi.

Ma'lumki,  $m/n$  oddiy (sonli) kasrda maxraj suratdan katta, ya'ni  $n > m$  bo'lsa, bu kasr to'g'ri,  $n \leq m$  holda esa noto'g'ri kasr deyiladi. Bu tushuncha ratsional kasrlar uchun quyidagicha kiritiladi.

**2-TA'RIF:** Agar (2) ratsional kasrda maxrajning darajasi  $n > m$  bo'lsa, u **to'g'ri**,  $n \leq m$  holda esa **noto'g'ri ratsional kasr** deb aytiladi.

Masalan,

$$R(x) = \frac{3x^4 - x^3 + 2x^2 - 5x + 1}{x^6 + 2x^4 + 3x^2 + 7}$$

to'g'ri,

$$R_1(x) = \frac{2x^3 - 3x^2 + 4x - 5}{x^3 + x^2 - 6x + 1}, \quad R_2(x) = \frac{2x^5 - 3x^3 + x + 5}{x^3 + 2x^2 - 4x + 1}$$

noto'g'ri ratsional kasrlar bo'ladi.

Har qanday noto'g'ri  $m/n$  ( $m > n$ ) oddiy kasrni

$$\frac{m}{n} = k + \frac{r}{n}, \quad k \in \mathbb{Z}, \quad r < n$$

ko'rinishda, ya'ni butun son va to'g'ri kasr yig'indisi kabi ifodalash mumkin. Xuddi shunday tasdiq noto'g'ri ratsional kasrlar uchun ham o'rinli bo'ladi, ya'ni ular uchun ushbu tenglikni hosil qilish mumkin:

$$R(x) = \frac{Q_m(x)}{P_n(x)} = L_{m-n}(x) + \frac{G_r(x)}{P_n(x)}, \quad r < n. \quad (3)$$

Bunda  $L_{m-n}(x)$  va  $G_r(x)$  ko'rsatilgan tartibli ko'phadlar bo'ladi.

Demak, har doim noto'g'ri ratsional kasrni ko'phad (butun funksiya) va to'g'ri ratsional kasr yig'indisi kabi ifodalash mumkin.

Masalan,

$$R(x) = \frac{2x^4 - 3x^3 + 1}{x^2 + x - 2}$$

noto'g'ri ratsional kasr suratini maxrajiga ustun usulida bo'lib, uni

$$R(x) = \frac{2x^4 - 3x^3 + 1}{x^2 + x - 2} = 2x^2 - 5x + 9 + \frac{-19x + 19}{x^2 + x - 2}$$

ko'rinishga keltira olamiz.

Har qanday ko'phad darajali funksiyalarning algebraik yig'indisi sifatida oson integrallamadi va uning integrali yana ko'phaddan iborat, ya'ni elementar funksiya bo'ladi. Demak, (3)

tenglikka asosan, har qanday ratsional kasrni integrallash masalasi to‘g‘ri ratsional kasrni integrallash masalasiga olib keladi. Shu sababli kelgusida faqat to‘g‘ri ratsional kasrlarni integrallash bilan shug‘ullanamiz.

**2. Eng sodda ratsional funksiyalar va ularni integrallash.** Quyidagi ko‘rinishdagi to‘g‘ri ratsional kasrlarni qaraymiz:

$$I. R_I(x) = \frac{A}{x-a}, \quad II. R_{II}(x) = \frac{A}{(x-a)^k},$$

$$III. R_{III}(x) = \frac{Ax+B}{x^2+px+q}, \quad IV. R_{IV}(x) = \frac{Ax+B}{(x^2+px+q)^k}.$$

Bunda  $A, B, a, p, q$ –haqiqiy sonlar,  $k=2,3,4, \dots$ , va  $x^2+px+q$  kvadrat uchhad haqiqiy ildizlarga ega emas, ya‘ni uning diskriminanti  $D=p^2-4q<0$  deb olinadi.

**3-TA‘RIF:** Yuqorida kiritilgan  $R_I(x) - R_{IV}(x)$  mos ravishda I–IV tur **eng sodda ratsional kasrlar** deb ataladi.

Eng sodda ratsional kasrlarni integrallash masalasini qaraymiz.

I va II turdagi oddiy kasrlarni integrallash jadval integrallariga oson keltiriladi:

$$R_I(x)dx = \frac{A dx}{x-a} = A \frac{d(x-a)}{x-a} = A \ln|x-a| + C;$$

$$R_{II}(x)dx = \frac{A dx}{(x-a)^k} = A (x-a)^{-k} d(x-a) =$$

$$= A \frac{(x-a)^{-k+1}}{-k+1} + C = \frac{A}{(1-k)(x-a)^{k-1}} + C, \quad k = 2,3,4, \dots.$$

III turdagi eng sodda  $R_{III}(x)$  ratsional kasrning integralini hisoblash usuli oldingi paragrafda ( $I_3$  integral) ko‘rilgan edi. Shunday bo‘lsada, bayonimizni to‘liq bo‘lishi va hisoblashlarni so‘ngi nuqtasigacha yetkazish maqsadida, bu usulni biz qarayotgan

$$p^2 - 4q < 0 \quad q - \frac{p^2}{4} = \sigma^2 > 0$$

hol uchun yana bir marta eslatamiz:

$$R_{III}(x)dx = \frac{Ax+B}{x^2+px+q} dx = \frac{\frac{A}{2}(2x+p) - \frac{Ap}{2} + B}{x^2+px+q} dx =$$

$$= \frac{A}{2} \frac{(2x+p)dx}{x^2+px+q} + (B - \frac{Ap}{2}) \frac{dx}{x^2+px+q} =$$

$$= \frac{x^2 + px + q = t}{(2x + p)dx = dt} = \frac{A}{2} \frac{dt}{t} + (B - \frac{Ap}{2}) \frac{dx}{x^2 + px + q} =$$

$$= \frac{A}{2} \ln|x^2 + px + q| + (B - \frac{Ap}{2}) \frac{d(x + \frac{p}{2})}{(x + \frac{p}{2})^2 + \sigma^2} =$$

$$= \frac{A}{2} \ln|x^2 + px + q| - (B - \frac{Ap}{2}) \frac{1}{\sigma} \operatorname{arctg} \frac{x + \frac{p}{2}}{\sigma} + C .$$

Endi IV turdagi eng sodda  $R_{IV}(x)$  kasrning integralini hisoblaymiz:

$$R_{IV}(x)dx = \frac{Ax + B}{(x^2 + px + q)^k} dx = \frac{\frac{A}{2}(2x + p) + B - \frac{Ap}{2}}{(x^2 + px + q)^k} dx =$$

$$= \frac{A}{2} \frac{(2x + p)dx}{(x^2 + px + q)^k} + (B - \frac{Ap}{2}) \frac{d(x + \frac{p}{2})}{[(x + \frac{p}{2})^2 + \sigma^2]^k} = \frac{A}{2} I_k + (B - \frac{Ap}{2}) J_k .$$

Bu yerdagi

$$I_k = \frac{(2x + p)dx}{(x^2 + px + q)^k}, k = 2, 3, 4, \dots$$

$$J_k = \frac{d(x + \frac{p}{2})}{[(x + \frac{p}{2})^2 + \sigma^2]^k}, \sigma = \sqrt{q - \frac{p^2}{4}}, k = 2, 3, 4, \dots$$

integrallarni hisoblaymiz:

$$I_k = \frac{(2x + p)dx}{(x^2 + px + q)^k} = \frac{x^2 + px + q = t}{(2x + p)dx = dt} = \frac{dt}{t^k} =$$

$$= \frac{1}{(1 - k)t^{k-1}} + C = \frac{1}{(1 - k)(x^2 + px + q)^{k-1}} + C ;$$

$$J_k = \frac{d(x + \frac{p}{2})}{[(x + \frac{p}{2})^2 + \sigma^2]^k} = \frac{t = x + \frac{p}{2}, dt = dx}{(t^2 + \sigma^2)^k} = \frac{dt}{(t^2 + \sigma^2)^k} =$$

$$= \frac{1}{\sigma^2} \frac{t^2 + \sigma^2 - t^2}{(t^2 + \sigma^2)^n} dt = \frac{1}{\sigma^2} \frac{dt}{(t^2 + \sigma^2)^{k-1}} - \frac{1}{\sigma^2} \frac{t^2 dt}{(t^2 + \sigma^2)^k} .$$

Bu tenglikdagi oxirgi integralga bo‘laklab integrallash formulasini qo‘llaymiz. Buning uchun integral ostidagi ifodani

$$u = t, \quad dv = \frac{t dt}{(t^2 + \sigma^2)^k}$$

ko‘rinishda bo‘laklaymiz. Bu holda  $du=dt$  va

$$v = \int dv = \frac{t dt}{(t^2 + \sigma^2)^k} = \frac{1}{2} \frac{d(t^2 + \sigma^2)}{(t^2 + \sigma^2)^k} = \frac{1}{2(1-k)(t^2 + \sigma^2)^{k-1}}$$

bo‘lgani uchun , bo‘laklab integrallash formulasiga asosan, ushbu tenglikni hosil qilamiz:

$$\frac{t^2 dt}{(t^2 + \sigma^2)^k} = \frac{t}{2(1-k)(t^2 + \sigma^2)^{k-1}} - \frac{1}{2(1-k)} \frac{dt}{(t^2 + \sigma^2)^{k-1}} .$$

Natijada  $J_k$  integralni hisoblash uchun

$$J_k = \frac{1}{\sigma^2} \frac{dt}{(t^2 + \sigma^2)^{k-1}} - \frac{1}{\sigma^2} \frac{t^2 dt}{(t^2 + \sigma^2)^k} =$$

$$= \frac{1}{\sigma^2} \frac{dt}{(t^2 + \sigma^2)^{k-1}} + \frac{t}{2(k-1)\sigma^2(t^2 + \sigma^2)^{k-1}} - \frac{1}{2(k-1)\sigma^2} \frac{dt}{(t^2 + \sigma^2)^{k-1}} =$$

$$= \frac{1}{2(k-1)\sigma^2} \frac{t}{(t^2 + \sigma^2)^{k-1}} + (2k-3) \frac{dt}{(t^2 + \sigma^2)^{k-1}}$$

formulani hosil etamiz. Bu yerdan  $J_k$  integralni hisoblash uchun ushbu

$$J_k = \frac{dt}{(t^2 + \sigma^2)^k} = \frac{1}{2(k-1)\sigma^2} \left[ \frac{t}{(t^2 + \sigma^2)^{k-1}} + (2k-3)J_{k-1} \right] \quad (4)$$

rekurrent formula o‘rinli ekanligini ko‘ramiz. Bu rekurrent formula bo‘yicha  $J_k$  integralni hisoblash xuddi shu ko‘rinishdagi, ammo  $k$  parametrining qiymati bittaga kichik bo‘lgan  $J_{k-1}$

integralni hisoblashga olib keladi. O‘z navbatida  $J_{k-1}$  integralni hisoblash  $J_{k-2}$  integralga keltiriladi va bu jarayon quyidagi  $J_1$  jadval integrali hosil bo‘lguncha davom ettiriladi:

$$J_1 = \frac{dt}{t^2 + \sigma^2} = \frac{1}{\sigma} \operatorname{arctg} \frac{t}{\sigma} + C.$$

$J_k$  integral uchun hosil qilingan ifodaga  $t$  va  $\sigma$  o‘rniga ularning

$$t = x + \frac{p}{2}, \quad \sigma = \sqrt{q - \frac{p^2}{4}}$$

qiymatlarini qo‘yib, bu integral javobini topamiz.

Misol sifatida IV turdagi ratsional kasrning ushbu integralini hisoblaymiz:

$$\begin{aligned} I &= \frac{(x-1)dx}{(x^2+2x+3)^2} = \frac{\frac{1}{2}(2x+2) - 2}{(x^2+2x+3)^2} dx = \\ &= \frac{1}{2} \frac{(2x+2)dx}{(x^2+2x+3)^2} - 2 \frac{dx}{(x^2+2x+3)^2} = \\ &= -\frac{1}{2} \frac{1}{(x^2+2x+3)} - 2 \frac{dx}{(x^2+2x+3)^2} = -\frac{1}{2} \frac{1}{(x^2+2x+3)} - 2J_2. \quad (5) \end{aligned}$$

Bunda  $J_2$  quyidagi integralni ifodalaydi:

$$\begin{aligned} J_2 &= \frac{dx}{(x^2+2x+3)^2} = \frac{dx}{[(x+1)^2+2]^2} = \frac{t=x+1,}{dt=dx} = \\ &= \frac{dt}{(t^2+2)^2} = \frac{1}{2} \frac{t^2+2-t^2}{(t^2+2)^2} dt = \frac{1}{2} \frac{dt}{t^2+(\sqrt{2})^2} - \frac{1}{2} \frac{t^2 dt}{(t^2+2)^2} = \\ &= \frac{1}{2\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} - \frac{1}{2} \frac{t^2 dt}{(t^2+2)^2}. \end{aligned}$$

Oxirgi integralni yuqorida ko‘rsatilgan usulda bo‘laklab integrallaymiz:

$$\frac{t^2 dt}{(t^2+2)^2} = \begin{matrix} u=t, & dv = \frac{t dt}{(t^2+2)^2} \\ du = dt, & v = -\frac{1}{2(t^2+2)} \end{matrix} = -\frac{t}{2(t^2+2)} + \frac{1}{2} \frac{dt}{t^2+2} =$$

$$= -\frac{t}{2(t^2+2)} + \frac{1}{2\sqrt{2}} \operatorname{arctg} \frac{t}{2} + C = -\frac{x+1}{2(x^2+2x+3)} + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + C.$$

Demak ,

$$J_2 = \frac{1}{4\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + \frac{x+1}{4(x^2+2x+3)} + C.$$

$J_2$  integralning bu qiymatini  $I$  uchun hosil qilingan (5) tenglikka qo'yib, berilgan  $I$  integral javobini topamiz :

$$I = \frac{(x-1)dx}{(x^2+2x+3)^2} = -\frac{x+2}{2(x^2+2x+3)} - \frac{\sqrt{2}}{4} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + C.$$

### Foydalanilgan adabiyotlar ro'yhati.

1. Latipova, S. (2024). YUQORI SINFLARDA GEOMETRIYA MAVZUSINI O'QITISHDA YANGI PEDAGOGIK TEXNOLOGIYALAR VA METODLAR. SINKVEYN METODI, VENN DIAGRAMMASI METODLARI HAQIDA. Theoretical aspects in the formation of pedagogical sciences, 3(3), 165-173.
2. Latipova, S. (2024, February). SAVOL-JAVOB METODI, BURCHAKLAR METODI, DEBAT (BAHS) METODLARI YORDAMIDA GEOMETRIYANI O'RGANISH. In Международная конференция академических наук (Vol. 3, No. 2, pp. 25-33).
3. Latipova, S., & Sharipova, M. (2024). KESIK PIRAMIDA MAVZUSIDA FOYDALANILADIGAN YANGI PEDAGOGIK TEXNOLOGIYALAR. 6X6X6 METODI, BBB (BILARDIM, BILMOQCHIMAN, BILIB OLDIM) METODLARI HAQIDA. Current approaches and new research in modern sciences, 3(2), 40-48.
4. Latipova, S. (2024). 10-11 SINFLARDA STEREOMETRIYA OQITISHNING ILMIY VA NAZARIY ASOSLARI. Академические исследования в современной науке, 3(6), 27-35.
5. Latipova, S. (2024). HILFER HOSILASI VA UNI HISOBLASH USULLARI. Центральноазиатский журнал образования и инноваций, 3(2), 122-130.
6. Latipova, S. (2024). HILFER MA'NOSIDA KASR TARTIBLI TENGLAMALAR UCHUN KOSHI MASALASI. Development and innovations in science, 3(2), 58-70.
7. Latipova, S. (2024). KESIK PIRAMIDA TUSHUNCHASI. KESIK PIRAMIDANING YON SIRTINI TOPISH FORMULALARI. Models and methods in modern science, 3(2), 58-71.
8. Shahnoza, L. (2023, March). KASR TARTIBLI TENGLAMALARDA MANBA VA BOSHLANG'ICH FUNKSIYANI ANIQLASH BO'YICHA TESKARI MASALALAR. In "Conference on Universal Science Research 2023" (Vol. 1, No. 3, pp. 8-10).
9. qizi Latipova, S. S. (2024). CAPUTO MA'NOSIDAGI KASR TARTIBLI TENGLAMALARDA MANBA FUNKSIYANI ANIQLASH BO 'YICHA TO 'G 'RI MASALALAR. GOLDEN BRAIN, 2(1), 375-382.
10. Latipova, S. S. (2023). SOLVING THE INVERSE PROBLEM OF FINDING THE SOURCE FUNCTION IN FRACTIONAL ORDER EQUATIONS. Modern Scientific Research International Scientific Journal, 1(10), 13-23.

11. Latipova, S. (2024). GEOMETRIYADA EKSTREMAL MASALALAR. В DEVELOPMENT OF PEDAGOGICAL TECHNOLOGIES IN MODERN SCIENCES (Т. 3, Выпуск 3, сс. 163–172).
12. Latipova, S. (2024). EKSTREMUMNING ZARURIY SHARTI. В SOLUTION OF SOCIAL PROBLEMS IN MANAGEMENT AND ECONOMY (Т. 3, Выпуск 2, сс. 79–90).
13. Latipova, S. (2024). FUNKSIYANING KESMADAGI ENG KATTA VA ENG KICHIK QIYMATI. В CURRENT APPROACHES AND NEW RESEARCH IN MODERN SCIENCES (Т. 3, Выпуск 2, сс. 120–129).
14. Latipova, S. (2024). EKSTREMUMLARNING YUQORI TARTIBLI HOSILA YORDAMIDA TEKSHIRILISHI. IKKINCHI TARTIBLI HOSILA YORDAMIDA EKSTREMUMGA TEKSHIRISH. В SCIENCE AND INNOVATION IN THE EDUCATION SYSTEM (Т. 3, Выпуск 3, сс. 122–133).
15. Latipova, S. (2024). BIR NECHA O'ZGARUVCHILI FUNKSIYANING EKSTREMUMLARI. В THEORETICAL ASPECTS IN THE FORMATION OF PEDAGOGICAL SCIENCES (Т. 3, Выпуск 4, сс. 14–24).
16. Latipova, S. (2024). SHARTLI EKSTREMUM. В МЕЖДУРОДНАЯ КОНФЕРЕНЦИЯ АКАДЕМИЧЕСКИХ НАУК (Т. 3, Выпуск 2, сс. 61–70).
17. Latipova, S. (2024). KASR TARTIBLI HOSILALARGA BO'LGAN ILK QARASHLAR. В CENTRAL ASIAN JOURNAL OF EDUCATION AND INNOVATION (Т. 3, Выпуск 2, сс. 46–51).
18. Latipova, S. (2024). TURLI EKSTREMAL MASALALAR. BAZI QADIMIY EKSTREMAL MASALALAR. В CENTRAL ASIAN JOURNAL OF EDUCATION AND INNOVATION (Т. 3, Выпуск 2, сс. 52–57).
19. Latipova, S. (2024). FUNKSIYA GRAFIGINI YASASHDA EKSTREMUMNING QO'LLANILISHI. В CENTRAL ASIAN JOURNAL OF EDUCATION AND INNOVATION (Т. 3, Выпуск 2, сс. 58–65).
20. Latipova, S. (2024). BIRINCHI TARTIBLI HOSILA YORDAMIDA FUNKSIYANING EKSTREMUMGA TEKSHIRISH, FUNKSIYANING EKSTREMUMLARI. В CENTRAL ASIAN JOURNAL OF EDUCATION AND INNOVATION (Т. 3, Выпуск 2, сс. 66–72).
21. Sharipova, M., & Latipova, S. (2024). TAKRORIY GRUPPALASHLAR. Development of pedagogical technologies in modern sciences, 3(3), 134-142.
22. Shahnoza Latipova. (2024). THE STRAIGHT LINE AND ITS DIFFERENT DEFINITIONS. Multidisciplinary Journal of Science and Technology, 4(3), 771–780.
23. Sharipova, M., & Latipova, S. (2024). IKKI O'ZGARUVCHILI TENGLAMALAR SISTEMASI. Центральноеазиатский журнал образования и инноваций, 3(2 Part 2), 93-103.
24. Latipova, S. (2024). THE STRAIGHT LINE AND ITS DIFFERENT DEFINITIONS. Multidisciplinary Journal of Science and Technology, 4(3), 771-780.
25. Latipova, S. (2024). KO 'PO 'ZGARUVCHILI FUNKSIYALARNING TURLI TA'RIFLARI. PEDAGOG, 7(5), 618-626.
26. Latipova Shahnoza. (2024). FAZODA ANALITIK GEOMETRIYANING SODDA MASALALARI. МЕДИЦИНА, ПЕДАГОГИКА И ТЕХНОЛОГИЯ: ТЕОРИЯ И ПРАКТИКА, 2(9), 406–419.
27. Jalolov, T. S. (2024). РАЗВИТИЕ ТЕХНОЛОГИЙ ИСКУССТВЕННОГО ИНТЕЛЛЕКТА В САМОДВИЖАЮЩИХСЯ РОБОТАХ. Methods of applying innovative and digital technologies in the educational system, 1(2), 1-7.



28. Jalolov, T. S. (2024). ЭФФЕКТИВНОЕ ИСПОЛЬЗОВАНИЕ ТЕХНОЛОГИЙ ИСКУССТВЕННОГО ИНТЕЛЛЕКТА В ЭКОНОМИЧЕСКОМ МОДЕЛИРОВАНИИ. Methods of applying innovative and digital technologies in the educational system, 1(2), 27-32.