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INTEGRAL TENGLAMALARNI FREDHOLM METODI BILAN YECHISH. FREDHOLM MUNOSABATLARI

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Annotasiya: Differensial operator qatnashgan tenglamani yechish esa integral operatorli tenglamani yechish masalasiga keladi. Shunday ekan, biz ushbu magistrlik dissertatsiyasida integral operatorlar qatnashgan tenglamalarni yechish usullariga to'xtalamiz. Aniqrog'i uch tipdagi integral operatorlarning xos qiymatlarini va rezolventasini topishning uch xil usulini namoyish qilamiz. Shundan Fredholm va Fridrixs tipidagi operatorlarga ko'proq to'xtalamiz. Integral operatorlar va ular bilan bog'liq integral tenglamalar nazariyasi Nuemann, Volterra, Liuvill, Fredholm, Fridrixs, Hilbert va Shmidtlar tomonidan rivojlantirilgan

$L_2[a;b]$ fazoda berilgan Volterra, Fredholm, Fridrixs tipidagi integral operatorlarni, ya'ni

$$(Vf)(x) = \int_a^x K(x,t) f(t)dt ,$$

$$(Kf)(x) = \int_a^b K(x,t) f(t)dt ,$$

$$(Hf)(x) = u(x) f(x) + \lambda \int_a^b K(x,y) f(y)dy$$

operatorlarni va ular bilan bog'liq integral tenglamalarni qaraymiz. Butun magistrlik dissertatsiya ishi davomida biz $K(x,t)$ dan chegaralangan va o'lchovli hamda simmetriklik shartini talab qilamiz, ya'ni:

A) $K(x,t) = K(t,x) / 0$ va $R = [a,b] \quad [a,b]$ kvadratda aniqlangan, haqiqiy qiymatli chegaralangan, o'lchovli funksiya bo'lishini talab qilamiz.

Faraz qilaylik, Fredholm tipidagi integral operatorning $\mu \quad 0$ nuqtadagi rezolventasini topish talab qilingan bo'lsin, ya'ni

$$(K - \mu I) f(x) = \varphi(x) \qquad \int_a^b K(x,t) f(t)dt - \mu f(x) = \varphi(x)$$

tenglamani, yoki bu yerda $\lambda = \mu^{-1}$ va $g(x) = -\mu^{-1}\varphi(x)$ deb olsak, u holda

$$f(x) = \lambda \int_a^b K(x,t) f(t)dt + g(x) \qquad (0.1)$$

tenglamani yechish masalasi qo'yiladi.

Biz bu paragrafda

$$u(x) = f(x) + \lambda \int_a^b K(x,t)u(t)dt$$

integral tenglamaning Fredholm tomonidan berilgan yechish usulini bayon qilamiz. Odatda

$$\Delta(\lambda) = 1 + \sum_{n=1} (-1)^n \frac{\lambda^n}{n!} A_n, \qquad (1)$$

$$A_n = \int_a^b \cdots \int_a^b \begin{vmatrix} K(t_1,t_1) & K(t_1,t_2) \cdots & K(t_1,t_n) \\ K(t_1,t_2) & K(t_2,t_2) \cdots & K(t_2,t_n) \\ \vdots \quad \cdots & \vdots \quad \cdots & \vdots \\ K(t_n,t_1) & K(t_n,t_2) \cdots & K(t_n,t_n) \end{vmatrix} dt_1 dt_2 \cdots dt_n,$$

$$D(x,t;\lambda) = \lambda K(x,t) + \sum_{n=1} (-1)^n \frac{\lambda^{n+1}}{n!} B_n(x,t), \qquad (2)$$

$$B_n(x,t) = \int_a^b \cdots \int_a^b \begin{vmatrix} K(x,t) & K(x,t_1) \cdots & K(x,t_n) \\ K(t_1,t) & K(t_1,t_1) \cdots & K(t_1,t_2) \\ \vdots \quad \cdots & \vdots \quad \cdots & \vdots \\ K(t_n,t) & K(t_n,t_1) \cdots & K(t_n,t_n) \end{vmatrix} dt_1 \cdots dt_n$$

funksiyalarga $K(x,y)$ yadro orqali qurilgan (1) integral tenglamaga mos Fredholm determinanti va minori deb ataladi. Keyinchalik (1) integral tenglamaning yechimini topish jarayonida muhim ahamiyatga ega bo‘ladigan Fredholmning 2 ta fundamental munosobatini keltirib utamiz:

$$D(x,t;\lambda) - \lambda K(x,t)\Delta(\lambda) = \lambda \int_a^b K(s,t)D(x,s;\lambda)ds, \qquad (3)$$

$$D(x,t;\lambda) - \lambda K(x,t)\Delta(\lambda) = \lambda \int_a^b K(x,\tau)D(\tau,t;\lambda)d\tau. \qquad (4)$$

(1) integral tenglamaning Fredholm tomonidan berilgan yechimi hadlari algebraik determinantlar ko‘rinishida bo‘lgan qator shaklida tasvirlangan Fredholm determinanti va minori bilan uzviy bog‘liq. Ushbu qatorlarning yaqinlashishini, ularning umumiy hadlarini biror yo‘l bilan baholash orqali ko‘rsatiladi. Buning uchun biz quyidagi Adamar teoremasidan foydalanamiz.

Adamar teoremasining isboti quyidagi lemma yordamida isbotlanadi.

1-lemma. *Agar*

$$A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ & & \cdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

algebraik determinantning har bir a_{ik} hadi haqiqiy bo‘lib

$$\overset{n}{\underset{i=1}{}} |a_{ik}|^2 \quad 1, \quad k = 1, \cdots, n$$

tengsizlikni qanoatlantirsin, u holda

$$|A| \quad 1$$

tengsizlik o‘rinli.

Bu lemmaning isbotini biz keltirmaymiz, lekin $n = 2$ va $n = 3$ bo‘lgan hollardagi geometrik talqinini beramiz. Tekislikda bir uchi koordinata boshi $O(0;0)$ da qolgan uchlari $P_1(x_1; y_1)$, $P_2(x_2; y_2)$ hamda $P_3(x_3; y_3)$ nuqtalarda bo‘lgan parallelogrammning yuzini topish masalasi quyilgan bo‘lsin. Bu parallelogrammning yuzi

$$A = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

formula bilan hisoblanadi. Agar OP_1 va OP_2 bektorlar uzunliklari birga teng, ya'ni $x_1^2 + y_1^2 = x_2^2 + y_2^2 = 1$ bo‘lsa, u holda bu parallelogramning yuzi 1 dan oshmaydi. Xuddi shunday uch o‘lchamli fazoda $OP_1(x_1, y_1, z_1)$, $OP_2(x_2, y_2, z_2)$ va $OP_3(x_3, y_3, z_3)$ vektorlar yordamida hosil qilingan parallelepipedning hajmi

$$A = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & x_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

formula yordamida hisoblanadi. Ma'lumkim birlik $|OP_i| = x_i^2 + y_i^2 + z_i^2 = 1, i = 1,2,3$ vektorlar yordamida qurilgan parallelepipedning hajmi birdan oshmaydi. Hajm 1 ga teng bo‘lishi uchun vektorlarning ortogonal bo‘lishi zarur va yetarlidir.

1-teorema (Adamar). *Agar*

$$B = \begin{vmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{vmatrix}$$

algebraik determinantning har bir b_{ik} hadi haqiqiy bo‘lib

$$|b_{ik}| \quad M, \quad i = 1,\cdots,n, k = 1,\cdots,n$$

tengsizlikni qanoatlantirsin, u holda

$$|B| \quad M^n \sqrt{n^n}$$

tengsizlik o‘rinli.

Isbot. Quyidagi belgilashni kiritamiz:

$$b_{i1}^2 + b_{i2}^2 + \cdots + b_{in}^2 = s_i, \quad i = 1,2,\cdots,n.$$

Quyidagi ikkita hol bo‘lishi mumkin.

1-hol. s_i lardan bir yoki bir nechtasi nolga teng, masalan $s_i = 0.$ U holda barcha $k = 1,2,\cdots,n$ lar uchun $b_{ik} = 0$ bo‘lib, bundan esa determinantning bitta satr elementlari nol bo‘lganligi uchun bu determinant nolga tengligini, ya'ni $B = 0$ ni olamiz. Bu holda teorema tasdig‘i bajariladi.

2-hol. s_i lardan birortasi ham nolga teng emas. U holda ixtiyoriy $i = 1,2,\cdots,n$ uchun $s_i > 0$ o‘rinli. Endi quyidagi determinantni qaraymiz

$$\frac{B}{\sqrt{s_1 s_2 \cdots s_n}} = \begin{vmatrix} \frac{b_{11}}{\sqrt{s_1}} & \cdots & \frac{b_{1n}}{\sqrt{s_1}} & \\ \vdots & \vdots & \cdots & \vdots \\ \frac{b_{n1}}{\sqrt{s_n}} & \cdots & \frac{b_{1n}}{\sqrt{s_n}} & \end{vmatrix}$$

uning har bir satr elementlari uchun

$$(\frac{b_{i1}}{\sqrt{s_i}})^2 + (\frac{b_{i2}}{\sqrt{s_i}})^2 + \cdots + (\frac{b_{in}}{\sqrt{s_i}})^2 = 1, \quad i = 1,2,\cdots,n$$

tenglik o‘rinli, ya'ni lemma shartlarini qanoatlantiradi. Bundan esa

$$|B| \le \sqrt{s_1 s_2 \cdots s_n}$$

tengsizlikning o‘rinli ekanligini olamiz. Teorema shartiga asosan $|b_{ik}| \le M$ bo‘lgani uchun $s_i \le nM^2$ bo‘lib, bundan kerakli

$$|B| \le M^n \sqrt{n^n}$$

tengsizlikni olamiz. 1-teorema isbot bo‘ldi.

Ushbu teoremadan foydalanib $K(x,t)$ yadro $|K(x,t)| \le M$ tengsizlikni qanoatlantirsa, u holda unga mos (1) qator bilan aniqlanuvchi $\Delta(\lambda)$ Fredholm determinanti λ parametrning barcha qiymatlarida yaqinlashuvchi bo‘ladi. Agar biz (1) ni darajali qator sifatida qarasak, uning yaqinlashish radiusi $R = \infty$ bo‘ladi. Bundan $\Delta(\lambda)$ funksiyaning kompleks tekislikda analitik funksiya ekanligi kelib chiqadi. Xuddi shunday (2) qator bilan aniqlanuvchi $D(x,t;\lambda)$ Fredholm minori ham λ parametrning barcha qiymatlarida va $(x;y) \in [a;b]^2$ da absolyut va tekis yaqinlashuvchi bo‘ladi. Demak uning yig‘indisi bo‘lgan $D(x,y;\lambda)$ funksiya $(x;y)$ bo‘yicha chegaralangan va λ parametrning analitik funksiyasi ekanligi kelib chiqadi.

(1) integral tenglamaning Fredholm tomonidan berilgan yechimi quyidagi teoremalarda o‘z ifodasini topgan.

2-teorema. *Agar*

1) $K(x,t)$ *yadro* R *kvadratda chegaralangan o‘lchovli,*

2) $f(x)$ *funksiya* $[a,b]$ *kesmada chegaralangan o‘lchovli,*

3) $\Delta(\lambda) \ne 0$ *bo‘lsin u holda* (1) *integral tenglama*

$$u(x) = f(x) + \int_a^b \frac{D(x,t;\lambda)}{\Delta(\lambda)} f(t)dt \qquad (5)$$

formula bilan ifodalanuvchi yagona yechimga ega.

Isbot. Faraz qilaylik, (1) tenglama $u(x)$ yechimga ega bo‘lsin. Uni quyidagi ko‘rinishda yozib olamiz

$$u(t) = f(t) + \lambda \int_a^b K(t,s)u(s)ds. \qquad (6)$$

(6) tenglikni ikkala qismini $D(x,t;\lambda)$ ko‘paytirib $t-$ o‘zgaruvchi bo‘yicha a dan b gacha integrallaymiz, natijada

$$\int_a^b D(x,t;\lambda)u(t)dt = \int_a^b D(x,t;\lambda)f(t)dt + \lambda \int_a^b \int_a^b D(x,t;\lambda)K(t,s)u(s)dsdt \qquad (7)$$

tenglikni olamiz. Ikki karrali integral ostidagi ifoda t va s lar bo'yicha integrallanuvchi bo'lganligi uchun, Fubini teoremasiga (5-teoremaga qarang) ko'ra unda integrallash tartibini o'zgartirish mumkin. Uni quyidagicha yozamiz

$$\int_a^b u(s) \ \lambda \int_a^b K(t,s)D(x,t;\lambda)dt \ \ ds. \qquad (8)$$

(3) Fredholm fundamental munosobatiga ko'ra (7.8) ni quyidagicha yozish mumkin

$$\int_a^b \{D(x,s;\lambda) - \lambda\Delta(\lambda)K(x,s)\}u(s)ds.$$

Bu tenglikka ko'ra (10) tenglamani quyidagicha yozish mumkin

$$\int_a^b D(x,t;\lambda)u(t)dt = \int_a^b D(x,t;\lambda)f(t)dt + \int_a^b D(x,s;\lambda)u(s)ds - \lambda\Delta(\lambda)\int_a^b K(x,s)u(s)ds.$$

Agar biz

$$\int_a^b D(x,t;\lambda)u(t)dt = \int_a^b D(x,s;\lambda)u(s)ds$$

ayniyatni hisobga olsak oxirgi tenglikdan quyidagini olamiz:

$$\lambda\int_a^b K(x,t)u(t)dt = \frac{1}{\Delta(\lambda)}\int_a^b D(x,t;\lambda)f(t)dt.$$

$\lambda\int_a^b K(x,t)u(t)dt$ ning bu ifodasini (1) ga qo'yib

$$u(x) = f(x) + \frac{1}{\Delta(\lambda)}\int_a^b D(x,t;\lambda)f(t)dt$$

ni olamiz. Demak, (1) tenglamaning ixtiyoriy yechimi (5) ko'rinishga ega ekan. Bu 2-teoremani isbotlaydi. Δ

Bu teoremadan natija sifatida aytish mumkinki, agar $\Delta(\lambda) \ \ 0$ bo'lsa, u holda (1) integral tenglamaga mos bir jinsli integral tenglama faqat nol yechimga ega bo'ladi.

3-teorema. *Agar* $\Delta(\lambda_0) = 0$ *va* $D(x,t;\lambda_0)$ *aynan nol funksiya bo'lmasa, u holda shunday* $t_0 \ \ [a,b]$ *mavjudki,* $D(x,t_0;\lambda_0)$ *funksiya*

$$u(x) = \lambda_0\int_a^b K(x,t)u(t)dt \qquad (9)$$

tenglamaning aynan nolga teng bo'lmagan o'lchovli yechimi bo'ladi.

Isbot. (9) integral tenglamaning yechimini topish uchun barcha λ larda o'rinli bo'lgan Fredholmning ikkinchi fundamental munosobatidan foydalanamiz. Bu holda (4) munosobat

$$D(x,t;\lambda_0) = \lambda_0 \int_a^b K(x,s)D(s,t;\lambda_0)ds$$

ko'rinishni oladi. Bu munosobat barcha $t \quad [a,b]$ larda, xususan, $t = t_0$ bo'lganda ham o'rinli, ya'ni

$$D(x,t_0;\lambda_0) = \lambda_0 \int_a^b K(x,s)D(s,t_0;\lambda_0)ds.$$

Bu esa (9) tenglamada $u(x)$ ni $D(x,t_0;\lambda_0)$ ga almashtirilganiga teng. Bu esa $u(x) = D(x,t_0;\lambda_0)$ funksiya (9) integral tenglamaning yechimi ekanligini anglatadi. Yuqorida keltirilgan Adamar teoremasidan ko'rinadiki, $D(x,t;\lambda)$ funksiya barcha $x,t \quad [a,b]$ larda tekis yaqinlashuvchi va hadlari chegaralangan o'lchovli funksiya-lardan iborat qator yig'indisi sifatida chegaralangan o'lchovlidir. 3-teorema isbot bo'ldi.

7.1-ta'rif. *Agar biror $\lambda = \lambda_0$ uchun $\Delta(\lambda_0) = 0$ bo'lsa, λ_0 ga $K(x,t)$ yadroning xarakteristik soni deyiladi.* (9) *tenglamaning nolmas yechimi esa $K(x,t)$ yadroning λ_0 xarakteristik songa mos fundamental funksiyasi deyiladi.*

Agar $\lambda_0 -$ $K(x,t)$ yadroning xarakteristik soni bo'lsa, u holda $\mu = 1/\lambda_0$ soni (1) tenglik bilan aniqlangan K operatorning xos qiymati bo'ladi. $K(x,t)$ yadroning fundamental funksiyalari, K operatorning xos funksiyalari bo'ladi.

7.3-teoremada $D(x,t;\lambda_0)$ aynan nolga teng emas shartini $\Delta (\lambda_0) \quad 0$ shart bilan almashtirish mumkin. Buning ucnun biz barcha λ larda o'rinli bo'lgan quyidagi tenglikdan ([22] ga qarang) foydalanamiz

$$\int_a^b D(x,x;\lambda)dx = -\lambda \Delta (\lambda). \qquad (10)$$

Faraz qilaylik, $\Delta(\lambda_0) = 0$ va $\Delta (\lambda_0) \quad 0$ bo'lsin. Ma'limki ((1) ga qarang), $\Delta(0) = 1$ shuning uchun $\lambda_0 \quad 0$. Agar biz (10) formulada $\lambda = \lambda_0$ desak, uning o'ng tomoni noldan farqli bo'ladi, shunday ekan uning chap tomoni ham nolmas bo'ladi. Bundan $D(x,x;\lambda_0)$ aynan nolga teng emasligi va o'z navbatida $D(x,t;\lambda_0)$ ning ham aynan nolga teng emasligi kelib chiqadi. Agar $\Delta(\lambda_0) = 0$ bilan birgalikda $D(x,t;\lambda_0) \quad 0$ bo'lsa, u holda (9) bir jinsli tenglamaning nolmas yechimlarini topish uchun «*yuqori tartibli*» minorlarni qarashga to'g'ri keladi. Yuqori tartibli minorlarni kiritish uchun biz quyidagi belgilashlardan foydalanamiz

$$K \begin{matrix} s_1, s_2, \ldots, s_n \\ t_1, t_2, \ldots, t_n \end{matrix} = \begin{vmatrix} K(s_1,t_1) & K(s_1,t_2) & \cdots & K(s_1,t_n) \\ K(s_2,t_1) & K(s_2,t_2) & \cdots & K(s_2,t_n) \\ \vdots & \cdots & \cdots & \vdots \\ K(s_n,t_1) & K(s_n,t_2) & \cdots & K(s_n,t_n) \end{vmatrix} \qquad (8.1)$$

va

$$B_n \begin{matrix} x_1, x_2, \ldots, x_p \\ y_1, y_2, \ldots, y_p \end{matrix} = {}_{a}^{b} \cdots {}_{a}^{b} K \begin{matrix} x_1, \ldots, x_p, t_1, \ldots, t_n \\ y_1, \ldots, y_p, t_1, \ldots, t_n \end{matrix} dt_1 \cdots dt_n .$$

Xususan $n = 0$ da

$$B_0 \begin{matrix} x_1, x_2, \ldots, x_p \\ y_1, y_2, \ldots, y_p \end{matrix} = K \begin{matrix} x_1, x_2, \ldots, x_p \\ y_1, y_2, \ldots, y_p \end{matrix} .$$

U holda $\Delta(\lambda)$ ning $p -$ tartibli minori quyidagicha aniqlanadi

$$D \begin{matrix} x_1, x_2, \ldots, x_p, \lambda \\ y_1, y_2, \ldots, y_p, \lambda \end{matrix} = {}_{n=0} (-1)^n \frac{\lambda^{p+n}}{n!} B_n \begin{matrix} x_1, x_2, \ldots, x_p \\ y_1, y_2, \ldots, y_p \end{matrix} := D_p(x, y; \lambda) . \qquad (8.2)$$

Xususiy hol $p = 1$ da $D_1(x, y; \lambda) = D(x, y; \lambda)$. Ta'kidlash joizki, agar biror $i \quad j$ uchun $x_i = x_j$ bo‘lsa, u holda (8.1) tenglik bilan aniqlangan

$$K \begin{matrix} x_1, x_2, \ldots, x_p \\ y_1, y_2, \ldots, y_p \end{matrix}$$

determinantning $i -$ chi va $j -$ chi satrlari bir xil bo‘ladi va natijada

$$K \begin{matrix} x_1, x_2, \ldots, x_p \\ y_1, y_2, \ldots, y_p \end{matrix} \quad 0$$

bo‘ladi. Bundan $D_p(x, y; \lambda) \quad 0$ ekanligi kelib chiqadi. Xuddi shunday biror $i \quad j$ uchun $y_i = y_j$ bo‘lsa ham $D_p(x, y; \lambda) \quad 0$ bo‘ladi. Agar (8.1) tenglik bilan aniqlangan

$$K \begin{matrix} x_1, x_2, \ldots, x_p \\ y_1, y_2, \ldots, y_p \end{matrix}$$

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determinantda x_i bilan x_j ning o‘rnini almashtirsak (8.1) determinantda $i-$ chi va $j-$ chi satrlarning o‘rni almashadi, bu esa (8.1) determinantning ishorasini o‘zgartiradi. Bu xossa $p-$ tartibli monor $D_p(x,y;\lambda)$ uchun ham o‘rinli, ya’ni agar biz $p-$ tartibli minor

$$D\begin{pmatrix} x_1,x_2,\dots,x_p,\lambda \\ y_1,y_2,\dots,y_p,\lambda \end{pmatrix} := D_p(x,y;\lambda)$$

da ((8.2) formulaga qarang) x_i bilan x_j ni o‘rnini almashtirsak, $p-$ tartibli minor $D_p(x,y;\lambda)$ ning faqat ishorasi almashadi.

Fredholmning umumlashgan fundamental munosabatlari quyidagilar:

$$D\begin{pmatrix} x_1,x_2,\dots,x_p,\lambda \\ y_1,y_2,\dots,y_p,\lambda \end{pmatrix} = \sum_{\alpha=1}^{p}(-1)^{\alpha+\beta}\lambda K(x_\alpha,y_\beta)\, D\begin{pmatrix} x_1,\dots,x_{\alpha-1},x_{\alpha+1},\dots,x_p,\lambda \\ y_1,\dots,y_{\beta-1},y_{\beta+1},\dots,y_p,\lambda \end{pmatrix} +$$

$$+\lambda\int_a^b K(t,y_\beta)\, D\begin{pmatrix} x_1,\dots,x_{\alpha-1},x_\alpha,x_{\alpha+1},\dots,x_p,\lambda \\ y_1,\dots,y_{\beta-1},t,y_{\beta+1},\dots,y_p,\lambda \end{pmatrix} dt, \qquad (8.3)$$

$$D\begin{pmatrix} x_1,x_2,\dots,x_p,\lambda \\ y_1,y_2,\dots,y_p,\lambda \end{pmatrix} = \sum_{\beta=1}^{p}(-1)^{\alpha+\beta}\lambda K(x_\alpha,y_\beta)\, D\begin{pmatrix} x_1,\dots,x_{\alpha-1},x_{\alpha+1},\dots,x_p,\lambda \\ y_1,\dots,y_{\beta-1},y_{\beta+1},\dots,y_p,\lambda \end{pmatrix} +$$

$$+\lambda\int_a^b K(x_\alpha,t)\, D\begin{pmatrix} x_1,\dots,x_{\alpha-1},t,x_{\alpha+1},\dots,x_p,\lambda \\ y_1,\dots,y_{\beta-1},y_\beta,y_{\beta+1},\dots,y_p,\lambda \end{pmatrix} dt. \qquad (8.4)$$

Yuqorida keltirilgan (7.10) munosabat quyidagi umumiy munosabatning

xususiy holidir

$$\int_a^b\cdots\int_a^b D\begin{pmatrix} x_1,x_2,\dots,x_p,\lambda \\ x_1,x_2,\dots,x_p,\lambda \end{pmatrix} dx_1 dx_2\cdots dx_p = (-1)^p\lambda^p\Delta^{(p)}(\lambda). \qquad (8.5)$$

(8.3)-(8.5) tengliklarning isboti [22] da keltirilgan. Faraz qilaylik, λ_0 soni $\Delta(\lambda)=0$ tenglamaning ildizi bo‘lsin. Ma'lumki, $\Delta(0)=1$ shuning uchun $\lambda_0 \neq 0$. $\Delta(\lambda)$ analitik funksiya bo‘lganligi uchun λ_0 uning chekli r karrali noli bo‘ladi, ya'ni

$$\Delta(\lambda_0)=0,\quad \Delta'(\lambda_0)=0,\quad \dots,\quad \Delta^{(r-1)}(\lambda_0)=0,\quad \Delta^{(r)}(\lambda_0)\neq 0.$$

Agar biz (8.5) formulada $\lambda=\lambda_0$ va $p=r$ desak, u holda (8.5) ning o‘ng tomoni nolmas bo‘ladi. Demak, uning chap tomoni ham nolmas, bu esa o‘z navbatida $p-$ tartibli $D_p(x,x;\lambda_0)$ minorning aynan nolmas ekanligini keltirib chiqaradi. Bu yerdan $D_p(x,y;\lambda_0)$

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ning aynan nol funksiya emasligi kelib chiqadi. Agar λ_0 soni $\Delta(\lambda)$ funksiyaning r karrali noli bo‘lsa, u holda shunday $q \quad r$ natural son mavjudki, quyidagilar bajariladi:

$$\Delta(\lambda_0)=0,\ D(x,y;\lambda_0) \quad 0,\ldots,D_{q-1}(x,y;\lambda_0) \quad 0$$

bo‘lib $D_q(x,y;\lambda_0)$ aynan nolmas bo‘ladi.

8.1-ta’rif. *Yuqorida aniqlangan* q *soniga* λ_0 *xarakteristik sonning karrasi deyiladi.*

Shuni ta’kidlaymizki, simmetrik yadrolar uchun $q=r$ tenglik o‘rinli. Xususan bizning holimizda ham $q=r$ bo‘ladi.

$D_q(x,y;\lambda_0)$ aynan nolmas funksiya bo‘lganligi uchun shunday $x_1=x_1$, $x_2=x_2,\ldots,x_q=x_q$, $y_1=y_1$, $y_2=y_2,\ldots,y_q=y_q$ nuqtalar mavjud bo‘lib,

$$D\begin{matrix} x_1,x_2,\ldots,x_q,\lambda_0 \\ y_1,y_2,\ldots,y_q,\lambda_0 \end{matrix} \quad 0$$

bo‘ladi. Endi Fredholmning (8.4) umumlashgan fundamental munosa-batida $\lambda=\lambda_0$, $p=q$ va

$$x_1=x_1,\ \ldots,\ x_{\alpha-1}=x_{\alpha-1},\ x_\alpha=x,\ x_{\alpha+1}=x_{\alpha+1},\ldots,x_q=x_q,$$
$$y_1=y_1,\ \ldots,\ y_{\alpha-1}=y_{\alpha-1},\ y_\alpha=y_\alpha,\ y_{\alpha+1}=y_{\alpha+1},\ldots,y_q=y_q$$

desak, quyidagi tenglikka ega bo‘lamiz

$$D\begin{matrix} x_1,\ldots,x_{\alpha-1},x,\ x_{\alpha+1},\ldots,x_q,\lambda \\ y_1,\ldots,y_{\alpha-1},y_\alpha,y_{\alpha+1},\ldots,y_q,\lambda \end{matrix} =$$

$$=\lambda_0 \int_a^b K(x,t)\, D\begin{matrix} x_1,\ldots,x_{\alpha-1},t,\ x_{\alpha+1},\ldots,x_q,\ \lambda_0 \\ y_1,\ldots,y_{\beta-1},y_\beta,y_{\beta+1},\ldots,y_q,\lambda_0 \end{matrix}\, dt\,. \qquad (8.6)$$

(8.6) tenglikning ikkala qismini noldan farqli bo‘lgan

$$D\begin{matrix} x_1,x_2,\ldots,x_q,\lambda_0 \\ y_1,y_2,\ldots,y_q,\lambda_0 \end{matrix} := D_q(x\,,y\,;\lambda_0)$$

bo‘lamiz va

$$\varphi_\alpha(x,\lambda_0)=\frac{D\begin{matrix} x_1,\dots,x_{\alpha-1},\, x,\, x_{\alpha+1},\dots,x_q,\ \lambda_0 \\ y_1,\dots,y_{\beta-1},y_\beta,y_{\beta+1},\dots,y_q,\lambda_0\end{matrix}}{D_q(x\,,y\,;\lambda_0)} \qquad (8.7)$$

belgilash kiritib, barcha $\alpha=1,2,\dots,q$ larda quyidagiga ega bo‘lamiz

$$\varphi_\alpha(x,\lambda_0)=\lambda_0\int_a^b K(x,t)\varphi_\alpha(t,\lambda_0)\,dt\,. \qquad (8.8)$$

(8.8) tenglik $\varphi_1(x,\lambda_0),\ \varphi_2(x,\lambda_0),\ \dots,\ \varphi_p(x,\lambda_0)$ lar bir jinsli (7.9) tenglamaning yechimlari ekanligini bildiradi. Bu yechimlar chegaralangan, o‘lchovli va (8.7) ga ko‘ra

$$\varphi_\alpha(x_\beta,\lambda_0)=\begin{matrix}1, & agar \quad \alpha=\beta \\ 0, & agar \quad \alpha \quad \beta.\end{matrix} \qquad (8.9)$$

8.1-lemma. *Bir jinsli (7.9) tenglamaning yechimlari sistemasi* $\varphi_1(x,\lambda_0),\ \varphi_2(x,\lambda_0),\ \dots,\ \varphi_q(x,\lambda_0)$ *chiziqli erklidir.*

Isbot. Faraz qilaylik,

$$C_1\,\varphi_1(x,\lambda_0)+C_2\,\varphi_2(x,\lambda_0)+C_q\,\varphi_q(x,\lambda_0)=0$$

tenglik biror $C_1,C_2,\dots,C_q$ sonlar uchun o‘rinli bo‘lsin. So‘nggi tenglikda $x=x_\alpha$ desak, (8.9) ga ko‘ra $C_\alpha=0,\ \alpha=1,2,\dots,q$ ga ega bo‘lamiz. Δ

Ma'lumki bir jinsli tenglama yechimlari yig‘indisi va songa ko‘paytmasi yana yechim bo‘ladi. Shuning uchun

$$u(x)=C_1\,\varphi_1(x,\lambda_0)+C_2\,\varphi_2(x,\lambda_0)+C_q\,\varphi_q(x,\lambda_0) \qquad (8.10)$$

funksiya ixtiyoriy $C_1,C_2,\dots,C_q$ sonlar uchun (7.9) bir jinsli tenglamaning yechimi bo‘ladi. Endi (7.9) bir jinsli tenglamaning ixtiyoriy yechimi (8.10) ko‘rinishga ega ekanligini ko‘rsatamiz. Faraz qilaylik, $v(x)$ bir jinsli (7.9) tenglamaning biror yechimi bo‘lsin., ya'ni

$$v(t)-\lambda_0\int_a^b K(t,s)v(s)\,ds \quad 0 \qquad (8.11)$$

bo‘lsin. U holda ixtiyoriy $H(x,t)$ uzluksiz funksiya uchun quyidagi ayniyat o‘rinli

$$\int_a^b v(t)H(x,t)-\lambda_0\int_a^b K(t,s)v(s)H(x,t)\,ds\ \,dt \quad 0. \qquad (8.12)$$

(8.11) dan (8.13) ni ayirib, quyidagiga ega bo‘lamiz

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$$v(x)=\int_a^b N(x,t)\,v(t)\,dt \tag{8.13}$$

bu yerda

$$N(x,t)=\lambda_0 K(x,t)-H(x,t)+\lambda_0\int_a^b K(s,t)\,H(x,s)\,ds\,.$$

Endi Fredholmning (8.3) umumlashgan fundamental munosabatida $\lambda=\lambda_0,\ p=q+1$ va $x_{q+1}=x,\ \ y_{q+1}=y$ desak va x_i bilan x_j ning o‘rni almashganda $D_p(x,y;\lambda_0)$ ning ishorasi almashinishini hisobga olsak, quyidagiga ega bo‘lamiz

$$D\begin{pmatrix}x,x_1,\dots,x_q,\lambda_0\\ y,y_1,\dots,y_q,\lambda_0\end{pmatrix}=\lambda_0 K(x,y)\,D\begin{pmatrix}x_1,\dots,x_\alpha,\dots,x_q,\lambda_0\\ y_1,\dots,y_\beta,\dots,y_q,\lambda_0\end{pmatrix}-$$

$$-\sum_{\alpha=1}^{q}\lambda_0 K(x_\alpha,y)\,D\begin{pmatrix}x_1,\dots,x_{\alpha-1},\,x,\,x_{\alpha+1},\dots,\,x_q,\,\lambda_0\\ y_1,\dots,y_{\alpha-1},y_\alpha,y_{\alpha+1},\dots,y_q,\lambda_0\end{pmatrix}+$$

$$+\lambda_0\int_a^b K(s,y)\,D\begin{pmatrix}x,x_1,\dots,x_q,\lambda_0\\ s,y_1,\dots,y_q,\lambda_0\end{pmatrix}ds\,. \tag{8.14}$$

(8.14) tenglikda

$$x_1=x_1,\ \dots,\ x_q=x_q,\quad y=t,\ y_1=y_1,\ \dots,\ y_q=y_q$$

almashtirish qilamiz, hamda (8.14) tenglikning ikkala qismini noldan farqli bo‘lgan

$$D\begin{pmatrix}x_1,x_2,\dots,x_q,\lambda_0\\ y_1,y_2,\dots,y_q,\lambda_0\end{pmatrix}:=D_q(x\,,y\,;\lambda_0)$$

ga bo‘lamiz va

$$H(x,y)=\frac{D\begin{pmatrix}x,x_1,\dots,x_q,\ \lambda_0\\ y,y_1,\dots,y_q,\lambda_0\end{pmatrix}}{D_q(x\,,y\,;\lambda_0)} \tag{8.15}$$

belgilash kiritib quyidagiga ega bo‘lamiz:

$$\sum_{\alpha=1}^{q}\lambda_0 K(x_\alpha,t)\varphi_\alpha(x,\lambda_0)=\lambda_0 K(x,t)-H(x,t)+\lambda_0\int_a^b K(s,t)\,H(x,s)\,ds\,. \tag{8.16}$$

(8.16) tenglikning o‘ng tomoni $N(x,t)$ ga teng. (8.12) aytiyat ixtiyoriy $H(x,t)$ uzluksiz funksiya uchun o‘rinli edi. Shuning uchun biz uni (8.15) tenglik bilan aniqlangan $H(x,t)$ bilan almashtiramiz. Natijada

$$N(x,t)=\sum_{\alpha=1}^{q}\lambda_0 K(x_\alpha,t)\varphi_\alpha(x,\lambda_0)$$

tenglikni olamiz. $N(x,t)$ ning bu ifodasini (8.13) tenglikning o‘ng tomoniga qo‘yib

$$v(x)=\lambda_0\sum_{\alpha=1}^{q}\varphi_\alpha(x,\lambda_0)\int_a^b K(x_\alpha,t)v(t)\,dt$$

tenglikka ega bo‘lamiz. Bundan $v(x)$ ning (8.10) ko‘rinishda tasvir-lanishi kelib chiqadi. Shunday qilib, biz Fredholmning ikkinchi fundamental teoremasini isbotladik.

8.1-teorema. *Agar* $\lambda=\lambda_0$ *soni* $K(x,t)$ *yadroning* q *karrasi xarakteristik soni bo‘lsa, u holda* (7.9) *bir jinsli tenglama* q *ta chiziqli bog‘lanmagan* $\varphi_\alpha(x,\lambda_0),\ \alpha=1,2,\dots,q$ *yechimlarga ega bo‘ladi va ixriyoriy* $u(x)$ *yechim ularning chiziqli kombinatsiyasi ko‘rinishida tasvirlanadi, ya'ni* $u(x)$ *yechim uchun* (8.10) *tenglik o‘rinli.*

Bu chiziqli bog‘lanmagan $\varphi_\alpha(x,\lambda_0),\ \alpha=1,2,\dots,q$ yechimlar sistemasi (8.7) tenglik bilan aniqlanadi.

Biz dastlab integral tenglamalarni Fredholm usuli bilan yechishga doir misollar qaraymiz.

11.1-misol. $L_2[-\pi,\pi]$ fazoda

$$u(x)=f(x)+\lambda\int_{-\pi}^{\pi}(1+\cos x\cos y)u(y)\,dy$$

integral tenglamaga mos Fredholm determinanti va Fredholm minorini toping.

Yechish. Bu integral tenglamaning yadrosi $K(x,y)=1+\cos x\cos y$ haqiqiy qiymatli va simmetriklik shartini qanoatlantiradi, ya'ni $K(x,y)=K(y,x)$. Endi (7.1) formula yordamida $A_n,\ n\in\mathbf{N}$ koeffitsiyentlarni hisoblaymiz:

$$A_1=\int_{-\pi}^{\pi}K(x,x)\,dx=\int_{-\pi}^{\pi}(1+\cos^2 x)\,dx=2\pi+\pi=3\pi.$$

Xuddi shunday A_2 koeffitsiyent hisoblanadi:

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$$A_2 = \int_{-\pi}^{\pi} dx \int_{-\pi}^{\pi} \begin{vmatrix} K(x,x) & K(x,y) \\ K(y,x) & K(y,y) \end{vmatrix} dy = \int_{-\pi}^{\pi} dx \int_{-\pi}^{\pi} \left[(1+\cos^2 x)(1+\cos^2 y) - (1+\cos x \cos y)^2\right] dy =$$

$$= \int_{-\pi}^{\pi} dx \int_{-\pi}^{\pi} \left(\cos^2 x + \cos^2 y - 2\cos x \cos y\right) dy = 2\pi^2 + 2\pi^2 - 0 = 4\pi^2 .$$

Intrgral tenglama yadrosining rangi 2 bo‘lganligi uchun, barcha $n \quad 3$ larda $A_n = 0$ bo‘ladi. Shuning uchun determinant $\Delta(\lambda)$ quyidagiga teng bo‘ladi:

$$\Delta(\lambda) = 1 - \lambda A_1 + \frac{1}{2}\lambda^2 A_2 = 1 - 3\pi\lambda + \frac{1}{2}\lambda^2 \quad 4\pi^2 = (\pi\lambda - 1)(2\pi\lambda - 1) . \qquad (11.1)$$

Integral tenglama yadrosining rangi 2 bo‘lganligi uchun, barcha $n \quad 2$ larda $B_n(x,t) = 0$ tenglik o‘rinli. $B_1(x,t)$ uchun esa quyidagi

$$B_1(x,t) = \int_{-\pi}^{\pi} \begin{vmatrix} K(x,t) & K(x,t_1) \\ K(t_1,t) & K(t_1,t_1) \end{vmatrix} dt_1 = (2\pi + \pi)(1 + \cos x \cos t) - 2\pi - \pi \cos x \cos t =$$

$$= \pi + 2\pi \cos x \cos t .$$

tenglik o‘rinli. Shunday qilib $D(x,t;\lambda)$ uchun quyidagiga ega bo‘lamiz:

$$D(x,t;\lambda) = \lambda K(x,t) - \lambda^2 B_1(x,t) = \lambda(1 + \cos x \cos t) - \lambda^2 (\pi + 2\pi \cos x \cos t) =$$

$$= \lambda(1 - \pi\lambda) + \lambda(1 - 2\pi\lambda) \cos x \cos t . \qquad (9.2)$$

9.2-misol. $K(x,y) = 1 + \cos x \cos y$ yadroning xarakteristik sonlari va fundamental funksiyalarini toping.

Yechish. Yadroning xarakteristik sonlari bu $\Delta(\lambda)$ ning nollaridir. 9.1-misolda $K(x,y) = 1 + \cos x \cos y$ yadroga mos Fredholm determinanti topilgan. Uning nollari ((9.1) ga qarang) $\lambda_1 = \pi^{-1}$ va $\lambda_2 = (2\pi)^{-1}$ lardir. Demak, ular $K(x,y)$ yadroning xarakteristik sonlari bo‘ladi. Bu λ_1 va λ_2 nuqtalarda birinchi tartibli minor $D(x,t;\lambda)$ noldan farqli bo‘lganligi uchun bu xarakteristik sonlarning karraliklari birga teng, ya'ni bir jinsli tenglamaning yechimlari to‘plami bir o‘lchamli chiziqli fazodir. (11.2) ga ko‘ra bu xarakteristik sonlarga mos keluvchi fundamental funksiyalar quyidagicha bo‘ladi:

$$\varphi(x,\lambda_1) = D(x,0;\lambda_1) = -\frac{1}{\pi}\cos x, \quad \psi(x,\lambda_2) = D(x,0;\lambda_2) = \frac{1}{4\pi} .$$

9.3-misol. $L_2[-\pi, \pi]$ fazoda

$$u(x)=\sin x+\lambda\int_{-\pi}^{\pi}(1+\cos x\cos y)u(y)dy \qquad (9.3)$$

Integral tenglamani qaraymiz. Bir jinslimas integral tenglamani $\lambda=\lambda_1=\pi^{-1}$ bo‘lganda 9.1-teoremadan foydalanib yeching.

Yechish. Qaralayotgan integral tenglama yechimga ega bo‘lishi uchun ozod had $f(x)=\sin x$ (9.3) ga mos bir jinsli tenglamaning barcha yechimlariga ortogonal bo‘lishi zarur va yetarlidir. (9.3) ga mos bir jinsli tenglamaning umumiy yechimi 9.2-misol va 8.1-teoremaga ko‘ra $C\varphi(x,\lambda_1)=-\frac{C}{\pi}\cos x$ ko‘rinishda bo‘ladi. Bu holda ortogonallik sharti bajariladi. Haqiqatan ham,

$$C\int_{-\pi}^{\pi} f(x)\varphi(x,\lambda_1)\,dx=C\int_{-\pi}^{\pi}\sin x\left(-\frac{1}{\pi}\cos x\right)dx=0.$$

Endi (11.3) tenglamaning umumiy yechimini topish uchun biz (8.15) tenglik bilan aniqlanuvchi $H(x,t)$ funksiyani qurishimiz kerak. Buning uchun esa bizga (8.2) tenglik bilan aniqlanuvchi ikkinchi tartibli minor $D_2(x,t;\lambda_1)$ kerak bo‘ladi. Integral tenglama yadrosining rangi 2 bo‘lganligi uchun, barcha n 1 larda

$$B_n\begin{pmatrix}x_1,x_2\\ t_1,\ t_2\end{pmatrix}\quad 0$$

ayniyat o‘rinli. Bundan (8.2) ga ko‘ra

$$D\begin{pmatrix}x_1,x_2,\lambda_1\\ t_1,\ t_2,\lambda_1\end{pmatrix}=\lambda_1^2\,B_0\begin{pmatrix}x_1,x_2\\ t_1,\ t_2\end{pmatrix}$$

tenglikka kelamiz. Murakkab bo‘lmagan hisoblashlar shuni ko‘rsatadiki

$$B_0\begin{pmatrix}x_1,x_2\\ t_1,\ t_2\end{pmatrix}=\cos x_1\cos t_1+\cos x_2\cos t_2-\cos x_1\cos t_2-\cos x_2\cos t_1$$

tenglik o‘rinli. Natijada biz

$$D\begin{pmatrix}x_1,x_2,\lambda_1\\ t_1,\ t_2,\lambda_1\end{pmatrix}=\pi^{-2}\left(\cos x_1\cos t_1+\cos x_2\cos t_2-\cos x_1\cos t_2-\cos x_2\cos t_1\right)$$

tenglikni olamiz. U holda $H(x,t)$ quyidagiga teng bo‘ladi:

$$H(x,t)=\frac{D\begin{pmatrix}x,0,\lambda_1\\ t,\,0,\lambda_1\end{pmatrix}}{D(0,0;\lambda_1)}=-\frac{1}{\pi}\left(\cos x\cos t+1-\cos x-\cos t\right).$$

Endi (9.8) yordamida xususiy yechim $u_0(x)$ ni topamiz:

$$u_0(x)=f(x)+\int_{-\pi}^{\pi}H(x,s)f(s)ds=\sin x+0=\sin x\,.$$

9.1-teoremaga koʻra umumiy yechim

$$u(x)=u_0(x)+C\varphi(x,\lambda_1)=\sin x+C\cos x\,.$$

Integral tenglamalarni yuqorida keltirilgan usullar bilan yechishga doir misollar qarashda davom etamiz.

9.4-misol. Volterra tipidagi ($x\in[0,1]$)

$$u(x)=1+\lambda\int_0^x u(t)dt \qquad (9.4)$$

integral tenglamani ketma-ket oʻrniga qoʻyish usulida yeching.

Yechish. Bu integral operatorning yadrosi $K(x,t)$ quyidagiga teng:

$$K(x,t)=\begin{cases}1, & agar\quad 0\le t<x\le 1,\\ 0, & agar\quad 0\le x\le t\le 1.\end{cases}$$

Koʻrsatamizki, $K(x,t)$ kvadrati bilan integrallanuvchi funksiyadan iborat boʻladi. Haqiqatan ham

$$\int_0^1\int_0^1|K(x,t)|^2\,dxdt=\int_0^1 dx\int_0^x dt=\int_0^1 xdx=\frac{1}{2}.$$

Demak, $|\lambda|<2$ boʻlsa, bu integral tenglama uchun ketma-ket oʻrniga qoʻyish usulini qullash mumkin. Bu misolda $f(x)=1$ va $(K^n f)(x)$ larni hisoblaymiz

$$(Kf)(x)=\int_0^1 K(x,t)f(t)dt=\int_0^x dt=x,$$

$$(K^2 f)(x)=\int_0^1\int_0^1 K(x,y)K(y,t)f(t)dtdy=\int_0^1\int_0^1 K(x,y)K(y,t)dtdy=\int_0^x dy\int_0^y dt=\int_0^x ydy=\frac{x^2}{2}.$$

xuddi shunday $(K^3 f)(x)$ ni hisoblash mumkin.

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$$(K^3 f)(x) = \int_0^x dy \int_0^y dt \int_0^t ds = \int_0^x dy \int_0^y t dt = \int_0^x \frac{y^2}{2} dy = \frac{x^3}{3!}$$

va hokazo

$$(K^n f)(x) = \frac{x^n}{n!}.$$

Shunday qilib, qaralayotga (9.4) integral tenglama yechimi quyidagi ko‘rinishga ega ekan

$$u(x) = 1 + \lambda x + \frac{(\lambda x)^2}{2!} + \cdots + \frac{(\lambda x)^n}{n!} + \cdots = e^{\lambda x}. \qquad (9.5)$$

Osongina kursatish mumkinki, $u(x) = e^{\lambda x}$ funksiya istalgan λ uchun (9.4) tenglamani qanoatlantiradi.

Endi (9.4) integral tenglamani ketma-ket yaqinlashishlar usulida yechamiz. Ravshanki, dastlabki u_0 yaqinlashish sifatida biz ixtiyoriy funksiyani tanlashimiz mumkin. $u_0(x) = 0$ deb olamiz. U holda (11.4) da $u(x)$ o‘rniga u_0 ni quyib birinchi yaqinlashish $u_1(x)$ uchun $u_1(x) = 1$ ni olamiz. Endi $u(x)$ o‘rniga $u_1(x)$ ni qo‘ysak, $u_2(x) = 1 + \lambda x$ yaqinlashishni olamiz. Shu kabi

$$u_3(x) = 1 + \lambda \int_0^x u_2(t) dt = 1 + \lambda \int_0^x (1 + \lambda t) dt = 1 + \lambda x + \frac{1}{2}\lambda^2 x^2.$$

Bu jarayonni davom ettirib $n+1$ - qadamda

$$u_{n+1}(x) = 1 + \lambda x + \cdots + \frac{1}{(n-1)!}\lambda^{n-1} x^{n-1} + \frac{1}{n!}\lambda^n x^n$$

ni hosil qilamiz. Bu tenglikda $n \to$ da limitga o‘tsak

$$\lim_{n \to} u_n(x) = e^{\lambda x}$$

(9.4) integral tenglama yechimini olamiz.

Demak, ixtiyoriy λ R uchun (9.4) integral tenglamaga ketma-ket yaqinlashishlar usulini qo‘llash mumkin va hosil bo‘lgan $\{u_n(x)\}$ ketma-ketlik (9.4) integral tenglama yechimi bo‘lgan $u(x) = e^{\lambda x}$ ga yaqinlashadi.

9.5-misol. $L_2[a, b]$ fazoda

$$u(x) = f(x) + \lambda \int_a^b \varphi(x)\psi(t)u(t) dt \qquad (9.6)$$

integral tenglamani yeching. Bunda φ va ψ funksiyalar chegaralangan o‘lchovli bo‘lib

$$\int_a^b \varphi(t)\psi(t)dt = 0 \qquad (9.7)$$

shartni qatoatlantiradi.

Yechish. (9.6) integral tenglamani ketma-ket o‘rniga qo‘yish usulida yechamiz. Buning uchun $u(t) = f(t) + \lambda \int_a^b \varphi(t)\psi(s)u(s)ds$ ni (9.6) ning o‘ng tomoniga qo‘yamiz.

$$u(x) = f(x) + \lambda \int_a^b \varphi(x)\psi(t)[f(t) + \int_a^b \varphi(t)\psi(s)u(s)ds]dt =$$

$$= f(x) + \lambda\varphi(x)\int_a^b \psi(t)f(t)dt + \lambda^2\varphi(x)\int_a^b \varphi(t)\psi(t)dt \int_a^b \psi(s)u(s)ds.$$

Agar bunda (9.7) shartdan foydalansak oxirgi tenglikdan $u(x)$ uchun quyidagi ifodani yoza olamiz

$$u(x) = f(x) + \lambda\varphi(x)\int_a^b \psi(t)f(t)dt. \qquad (9.8)$$

Bu tenglikning o‘ng tomoni $u(x)$ ga bog‘liq emas, keyingi o‘rniga qo‘yishlar yana (9.8) tenglikka olib keladi. Demak ixtiyoriy $\lambda \in R$ uchun (9.6) integral tenglamaning yechimi (9.8) ko‘rinishda bo‘lar ekan.

Endi yuqorida keltirilgan (9.6) - integral tenglamani ketma-ket yaqinlashishlar usulida yechamiz.

Boshlang‘ich yaqinlashish sifatida $u_0(x) = f(x)$ ni olamiz. U holda birinchi yaqinlashish

$$u_1(x) = f(x) + \lambda\varphi(x)\int_a^b \psi(t)f(t)dt \qquad (9.9)$$

bo‘ladi. $u_1(x)$ ni (9.9) ning o‘ng tomoniga qo‘yib $u_2(x)$ uchun quyidagini olamiz

$$u_2(x) = f(x) + \lambda\varphi(x)\int_a^b \psi(t)[f(t) + \lambda\varphi(t)\int_a^b \psi(s)f(s)ds]dt =$$

$$= f(x) + \lambda\varphi(x)\int_a^b \psi(t)f(t)dt + \lambda^2\varphi(x)\int_a^b \varphi(t)\psi(t)dt \int_a^b \psi(s)f(s)ds. \qquad (9.10)$$

Ortogonallik sharti bo‘lgan (9.7) dan foydalanib (11.10) dan $u_2(x) = u_1(x)$ ga kelamiz. Xuddi shunday $u_n(x) = u_1(x),\ n \geq 3$ tenglikka kelamiz. Demak, biz (9.6) integral tenglamaga ketma-ket yaqinlashishlar usulini qo‘llab, ikkinchi hadidan boshlab o‘zgarmas bo‘lgan

$$u_n(x) = f(x) + \lambda\varphi(x) \int_a^b \psi(t) f(t)dt \qquad (9.11)$$

funksional ketma-ketlikka ega bo‘ldik. Bundan $\lim_{n\to} u_n(x) = u_1(x)$ ni olamiz.

Demak, istalgan $\lambda \in R$ da (9.6) tenglama yagona yechimga ega va u (9.9) tenglikning o‘ng tomoni bilan ifodalanar ekan.

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